4) If $A=\begin{bmatrix}0 & -x\\ x & 0\end{bmatrix}$, $B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ and $x^2=-1$, then show that

$(A+B)^2=A^2+B^2$

Solution: We have, $A=\begin{bmatrix}0 & -x\\ x & 0\end{bmatrix}$, $B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ and $x^2=-1$

$$\therefore (A+B)=\begin{bmatrix}0 & -x+1\\ x+1 & 0\end{bmatrix}$$

$$\therefore (A+B)^2=\begin{bmatrix}0 & -x+1\\ x+1 & 0\end{bmatrix}\begin{bmatrix}0 & -x+1\\ x+1 & 0\end{bmatrix}=\begin{bmatrix}1-x^2 & 0\\ 0 & 1-x^2\end{bmatrix} \quad -(1)$$

Also, $$A^2=A\cdot A=\begin{bmatrix}0 & -x\\ x & 0\end{bmatrix}\begin{bmatrix}0 & -x\\ x & 0\end{bmatrix}=\begin{bmatrix}-x^2 & 0\\ 0 & -x^2\end{bmatrix}$$

And. $$B^2=B\cdot B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\therefore A^2+B^2=\begin{bmatrix}-x^2 & 0\\ 0 & -x^2\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1-x^2 & 0\\ 0 & 1-x^2\end{bmatrix} \quad -(2)$$

From equations (1) and (2),

we have $(A+B)^2=A^2+B^2$