α-Decay

- (n/p ratio increases)

$$^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

Energy of products < Energy reactants
This loss in energy appears as kinetic energy of the products

γ-particle is released when Y is in excited state of nucleus.

$$^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y^{*} + {}^{4}_{2}He$$

$$^{A-4}_{Z-2}Y^{*} \longrightarrow {}^{A-4}_{Z-2}Y + \gamma$$

(γ Photon)

β-decay
(n/p decreases)

$$n \longrightarrow p + e^{-} + \bar{\nu}$$

($\bar{\nu}$ Anti-neutrino)

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + \beta^{-} + \bar{\nu}$$

Some energy is carried by anti-neutrino

Mean life

It is the average life of all the nuclei

Let no. of nuclei at $t=0$ be N_0 and at $t=t$ be N
Then

$$N = N_0 e^{-\lambda t}$$

Let no. of nuclei dissociated in time dt be dN

These dN particles have a life t as they existed uptill time, t, and not after that.

Sum of lives of these particles $= t + t + t + \dots$ (dN times) $= t\,dN$

Sum of lives of all particles $= \int_0^\infty t\,dN$

$$= -\lambda \int_0^\infty t N_0 e^{-\lambda t}\,dt$$

$$\int_0^\infty t e^{-\lambda t}\,dt = t\int_0^\infty e^{-\lambda t}\,dt - \int_0^\infty \frac{dt}{dt}\left(\int e^{-\lambda t}\,dt\right)dt$$

$$= \left[\frac{te^{-\lambda t}}{-\lambda}\right]_0^\infty - \int_0^\infty \frac{e^{-\lambda t}}{-\lambda}\,dt$$

$$= 0 - 0 + \frac{1}{\lambda^2}\left(e^{-\lambda t}\right)_0^\infty = -\frac{1}{\lambda^2}$$

Sum of lives of N_0 particles $= -\lambda \int_0^\infty t N_0 e^{-\lambda t}\,dt = -\lambda N_0 \times -\frac{1}{\lambda^2}$

$$= \frac{N_0}{\lambda}$$

Avg life $=$ Sum $/ N_0 = \frac{1}{\lambda} = \tau$

Radio activity

↳ Rate of emission is called radioactivity of the substance

↳ Radioactivity does not depend on external factors like temperature and pressure

↳ Radioactivity is independent of electronic configuration

↳ $-\frac{dN}{dt} = \lambda N$ (differential equation)

On solving, $N(t) = N_0 e^{-\lambda t}$

↳ Probability that a particle is not decayed at time t $= \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$

↳ Half-life, $t_{1/2} = \frac{0.693}{\lambda}$ where λ is decay constant

↳ Activity of the substance $= \lambda N$