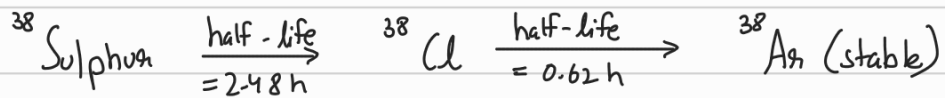
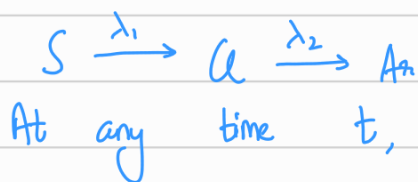


Q5. Sometimes a radioactive nucleus decays into a nucleus which itself is radioactive. An example is:



Assume that we start with 1000 ${}^{38}\text{S}$ nuclei at time $t=0$. The number of ${}^{38}\text{Cl}$ is of count zero at $t=0$ and will again be zero at $t=\infty$. At what value of t , would the number of counts be a maximum.



$$\frac{dN_{Cl}}{dt} \text{ for } Cl = \lambda_1 N_S - \lambda_2 N_{Cl} \quad \text{When } N_{Cl} \text{ is max} \Rightarrow \frac{dN_{Cl}}{dt} = 0$$

$$N_S = N_0 e^{-\lambda_1 t}$$

$$\frac{dN}{dt} = \lambda_1 N_0 e^{-\lambda_1 t} - \lambda_2 N$$

$$\frac{dN}{dt} + \lambda_2 N = \lambda_1 N_0 e^{-\lambda_1 t}$$

$$e^{\lambda_2 t} \frac{dN}{dt} + \lambda_2 e^{\lambda_2 t} N = \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t}$$

[Integrating factor = $e^{\lambda_2 t}$]

$$\frac{d(Ne^{\lambda_2 t})}{dt} = \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t}$$

$$Ne^{\lambda_2 t} = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} \left[e^{(\lambda_2 - \lambda_1)t} - 1 \right]$$

$$N_{Cl} = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

$$\frac{dN_{Cl}}{dt} = 0 \Rightarrow \lambda_1 N_S = \lambda_2 N_{Cl}$$

$$\Rightarrow \lambda_1 N_0 e^{-\lambda_1 t} = \frac{\lambda_2 \lambda_1 N_0}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

$$\begin{aligned} (\lambda_2 - \lambda_1) e^{-\lambda_1 t} &= \lambda_2 (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \\ -\lambda_1 e^{-\lambda_1 t} &= -\lambda_2 e^{-\lambda_2 t} \\ e^{(\lambda_2 - \lambda_1)t} &= \frac{\lambda_2}{\lambda_1} \end{aligned}$$

$$(\lambda_2 - \lambda_1) t = \ln\left(\frac{\lambda_2}{\lambda_1}\right)$$

$$t = \frac{1}{(\lambda_2 - \lambda_1)} \ln\left(\frac{\lambda_2}{\lambda_1}\right)$$

$$\lambda_1 = \frac{\ln 2}{2.48} \text{ h}^{-1}$$

$$\lambda_2 = \frac{\ln 2}{0.62} \text{ h}^{-1}$$

$$\Rightarrow t = \frac{1}{\ln 2 \left(\frac{1}{0.62} - \frac{1}{2.48} \right)} \ln\left(\frac{2.48}{0.62}\right) = \frac{\ln 4 \times 0.62 \times 2.48}{\ln 2 (2.48 - 0.62)} = 1.65 \text{ h}$$