

1. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is (2002)

- 1) $3x^2 - 19x + 3 = 0$ 2) $3x^2 + 19x - 3 = 0$
3) $3x^2 - 19x - 3 = 0$ 4) $x^2 - 5x + 3 = 0$

Ans.

(1) We have $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$;
 $\Rightarrow \alpha$ & β are roots of equation, $x^2 = 5x - 3$ or
 $x^2 - 5x + 3 = 0$

$$\therefore \alpha + \beta = 5 \text{ and } \alpha\beta = 3$$

Thus, the equation having $\frac{\alpha}{\beta}$ & $\frac{\beta}{\alpha}$ as its roots

$$\text{is } x^2 - x\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \frac{\alpha\beta}{\alpha\beta} = 0$$

$$\Rightarrow x^2 - x\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right) + 1 = 0 \text{ OR } 3x^2 - 19x + 3 = 0$$