Question 20: If $b_1 b_2 = 2 (c_1 + c_2)$, then at least one of the equations $x^2 + b_1x + c1 = 0$ and $x^2 + b_2x + C_2 = 0$ has _____ roots.

Solution:

Let D1 and D2 be discriminants of $x^2 + b_1x + c1 = 0$ and $x^2 + b_2x + C_2 = 0$, respectively.

Then,

$D_1 + D_2 = b_1^2 - 4c_1 + b_2^2 - 4c_2$

$= (b_1^2 + b_2^2) - 4 (c_1 + c_2)$

$= b_1^2 + b_2^2 - 2b_1b_2$ [Because $b_1b_2 = 2 (c_1 + c_2)$] $= (b_1 - b_2)^2 \geq 0$

$\Rightarrow D_1 \geq 0$ or $D_2 \geq 0$ or D_1 and D_2 both are positive.

Hence, at least one of the equations has real roots.