

Question 21: If the roots of the equation $x^2 + 2ax + b = 0$ are real and distinct and they differ by at most $2m$ then b lies in what interval?

Solution:

Let the roots be α, β

$$\alpha + \beta = -2a \text{ and } \alpha\beta = b$$

Given, $|\alpha - \beta| \leq 2m$

$$\text{or } |\alpha - \beta|^2 \leq (2m)^2 \text{ or}$$

$$(\alpha + \beta)^2 - 4\alpha\beta \leq 4m^2 \text{ or}$$

$$4a^2 - 4b \leq 4m^2$$

$$\Rightarrow a^2 - m^2 \leq b \text{ and discriminant } D > 0 \text{ or}$$

$$4a^2 - 4b > 0$$

$$\Rightarrow a^2 - m^2 \leq b \text{ and } b < a^2.$$

Hence, $b \in [a^2 - m^2, a^2)$.