If m is chosen in the quadratic equation

$$(m^2 + 1)\,x^2 - 3x + (m^2 + 1)^2 = 0$$

such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is:

(1) $10\sqrt{5}$ (2) $8\sqrt{3}$ (3) $8\sqrt{5}$ (4) $4\sqrt{3}$

(3) Sum of roots $= \dfrac{3}{m^2+1}$

$\because$ sum of roots is greatest. $\therefore\ m = 0$

Hence equation becomes $x^2 - 3x + 1 = 0$

Now, $\alpha+\beta = 3,\ \alpha\beta = 1 \Rightarrow |-\alpha-\beta| = \sqrt{5}$

$$\left|\alpha^3 - \beta^3\right| = \left|(\alpha-\beta)\left(\alpha^2+\beta^2+\alpha\beta\right)\right| = \sqrt{5}\,(9-1) = 8\sqrt{5}$$