

2) Solve system of linear equations, using matrix method.

$$2x + 3y + 3z = 5$$

$$x - 2y + z = -4$$

$$3x - y - 2z = 3$$

Solution:

The given system of equations can be written in the form

$$AX = B, \text{ where } A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

Now,

$$|A| = 2(4+1) - 3(-2-3) + 3(-1+6) = 2(5) - 3(-5) + 3(5) \\ = 10 + 15 + 15 = 40 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

$$\text{Now, } A_{11} = 5, A_{12} = 5, A_{13} = 9$$

$$A_{21} = 3, A_{22} = -13, A_{23} = 1$$

$$A_{31} = 9, A_{32} = 1, A_{33} = -7$$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj } A) = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 25 - 12 + 27 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Hence, $x=1, y=2$ and $z=-1$