

Q.1) Solve the system of linear equations, using matrix method.

$$x - y + z = 4$$

$$2x + y - 3z = 0$$

$$x + y + z = 2$$

solution: The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

Now,

$$|A| = 1(1+3) + 1(2+3) + 1(2-1) = 4 + 5 + 1 = 10 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

$$\text{Now, } A_{11} = 4, A_{12} = -5, A_{13} = 1$$

$$A_{21} = 2, A_{22} = 0, A_{23} = -2$$

$$A_{31} = 2, A_{32} = 5, A_{33} = 3$$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj}A) = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 16 + 0 + 4 \\ -20 + 0 + 10 \\ 4 + 0 + 6 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Hence, $x = 2$, $y = -1$, and $z = 1$.