

Question 3: Evaluate the Taylor Series for $f(x) = x^3 - 10x^2 + 6$ at $x = 3$.

Solution: First, we will find the derivatives of the given function.

$$f(x) = x^3 - 10x^2 + 6 \Rightarrow f(3) = -57$$

$$f'(x) = 3x^2 - 20x \Rightarrow f'(3) = 33$$

$$f''(x) = 6x - 20 \Rightarrow f''(3) = -2$$

$$f'''(x) = 6 \Rightarrow f'''(3) = 6$$

$$f^{(4)}(x) = 0$$

Therefore, the required series is:

$$\begin{aligned}x^3 - 10x^2 + 6 &= \sum_{n=0}^{\infty} \frac{f^{(n)}(3)}{n!} (x - 3)^n \\&= f(3) + f'(3)(x - 3) + \frac{f''(3)}{2!} (x - 3)^2 + \frac{f'''(3)}{3!} (x - 3)^3 + 0 \\&= -57 - 33(x - 3) - (x - 3)^2 + (x - 3)^3\end{aligned}$$

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