Question 2: EValuate the Taylor Series for $f(x) = \cos(x)$ for $x = 0$.

Solution: We need to take the derivatives of the cos x and evaluate them at $x = 0$.

$f(x) = \cos x \Rightarrow f(0) = 1$

$f'(x) = -\sin x \Rightarrow f'(0) = 0$

$f''(x) = -\cos x \Rightarrow f''(0) = -1$

$f'''(x) = \sin x \Rightarrow f'''(0) = 0$

$f''''(x) = \cos x \Rightarrow f''''(4) = 1$

$f(5)(x) = -\sin x \Rightarrow f(5)(0) = 0$

$f(6)(x) = -\cos x \Rightarrow f(6)(0) = -1$

Therefore, according to the Taylor series expansion;

$$\cos x = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$= f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \frac{f^{(5)}(0)}{5!}x^5 + \cdots$$

$$= 1 + 0 - \frac{1}{2!}x^2 + 0 + \frac{1}{4!}x^4 + 0 - \frac{1}{6!}x^6 + \cdots$$

$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \cdots$

$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$