Que 5: If $f(x)=(x^2-1)/(x^2+1)$, for every real number x, then the minimum value of f [IIT 1998]

A) Does not exist because f is unbounded

B) Is not attained even though f is bounded

C) Is equal to 1

D) Is equal to -1

Correct Answer: D

Solution :

$$f(x)=(x^2-1)/(x^2+1)=(x^2+1-2)/(x^2+1)=1-2/(x^2+1)$$

$$\therefore f(x)<1\ \forall x \text{ and } \geq -1 \text{ as } 2/(x^2+1)\leq 2$$

$$\therefore -1\leq f(x)<1$$

Hence f(x) has minimum value -1 and also there is no maximum value.

Alternative solution :

$$f'(x)=[\{(x^2+1)2x\}-\{(x^2-1)2x\}]/(x^2+1)^2=4x/(x^2+1)^2$$

$$f'(x)=0\Rightarrow x=0$$

$$f''(x)=[\{(x^2+1)^2\}4-4x\{2(x^2+1)2x\}]/(x^2+1)^4$$

$$=[\{(x^2+1)4\}-16x(x)]/(x^2+1)^3=(-12x^2+4)/(x^2+1)^3$$

$$\therefore f''(0)>0$$

∴There is only one critical point having minima.

Hence f(x) has least value at x=0.

$$f_{min}=f(0)=-1/1=-1.$$