Question 14: If $a^2 + b^2 + c^2 = 1$, then ab + bc + ac lies in the interval

(a) [½, 2]

(b) [-1, 2]

(c) [-1/2, 1]

(d) [-1, ½]

Solution:

Given that $a^2 + b^2 + c^2 = 1$ ...(i)

We know $(a+b+c)^2 \geq 0$

$a^2+b^2+c^2+2ab+2bc+2ac \geq 0$

$\Rightarrow 1 + 2(ab+bc+ac) \geq 0$ (from (i))

$\Rightarrow 2(ab+bc+ac) \geq -1$

$\Rightarrow (ab+bc+ac) \geq -\frac{1}{2}$ ...(ii)

We know that $\frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2] \geq 0$

$\Rightarrow a^2 + b^2 + c^2 - ab - bc - ac \geq 0$

$\Rightarrow ab + bc + ac \leq 1$ ...(iii)

Combining (ii) and (iii)

$-\frac{1}{2} \leq (ab+bc+ac) \leq 1$

Therefore $(ab+bc+ac) \in [-\frac{1}{2}, 1]$

Hence option c is the answer.