

Question 12: Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in A.P and $1/\alpha + 1/\beta = 4$, then the value of $|\alpha - \beta|$ is

(a) $\sqrt{34}/9$

(b) $2\sqrt{13}/9$

(c) $\sqrt{61}/9$

(d) $2\sqrt{17}/9$

Solution:

Given that α and β be the roots of equation $px^2 + qx + r = 0$

So sum of roots = $\alpha + \beta = -q/p$

Product of roots = $\alpha\beta = r/p$

Given $1/\alpha + 1/\beta = 4$

$$\Rightarrow (\alpha + \beta)/\alpha\beta = 4$$

$$\Rightarrow (\alpha + \beta) = 4\alpha\beta$$

$$\Rightarrow -q/p = 4r/p$$

$$\Rightarrow 4r = -q \dots(i)$$

p, q, r are in A.P.

$$\text{So } 2q = (p+r) \dots(ii)$$

Substitute (i) in (ii)

$$-8r = p+r$$

$$-9r = p$$

$$\Rightarrow r/p = -1/9$$

$$\Rightarrow \alpha\beta = -1/9$$

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

$$= (-4/9)^2 + 4/9$$

$$= 16/81 + 36/81$$

$$= 52/81$$

$$|\alpha - \beta| = \sqrt{52}/9$$

$$= 2\sqrt{13}/9$$

Hence option b is the answer.