

Question 7: Let $D = a^2 + b^2 + c^2$, a, b being consecutive integers and $c = ab$ then \sqrt{D} is

- (a) always an even integer
- (b) always an odd integer
- (c) sometimes an odd integer, sometimes not
- (d) sometimes a rational number, sometimes not

Solution:

Given that a and b are consecutive integers.

$$\text{So } b = a+1$$

$$\text{Also } c = ab$$

$$D = a^2 + b^2 + c^2$$

$$= a^2 + (a+1)^2 + a^2b^2$$

$$= a^2 + a^2 + 2a + 1 + a^2b^2$$

$$= 2a^2 + 2a + 1 + a^2b^2$$

$$= 2a(a+1) + 1 + a^2b^2$$

$$= 2ab + 1 + (ab)^2$$

$$= (ab+1)^2$$

$$\sqrt{D} = ab+1$$

$\Rightarrow \sqrt{D}$ is always an odd integer

(ab is even number since either a or b will be even. When we add 1 to it, we get an odd number.)

Hence option b is the answer.