Question 5: A value of b for which the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have one root in common is

(a) $-\sqrt{2}$

(b) $-i\sqrt{3}$

(c) $i\sqrt{5}$

(d) 2

Solution:

Given that $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have one root in common.

The formula to find the common root is given below.

If α is a common root of $a_1x^2+b_1x+c_1 = 0$ and $a_2x^2+b_2x+c_2 = 0$

Then

$\alpha^2/(b_1c_2-b_2c_1) = \alpha/(a_2c_1-a_1c_2) = 1/(a_1b_2-a_2b_1)$ ...(i)

Comparing with given equations, we get

$a_1 = 1, b_1 = b, c_1 = -1$

$a_2 = 1, b_2 = 1, c_2 = b$

$\alpha^2/(b_1c_2-b_2c_1) = \alpha/(a_2c_1-a_1c_2) = 1/(a_1b_2-a_2b_1)$

Substituting the values in (i) we get

$\alpha^2/(b^2 +1) = \alpha/(-1-b) = 1/(1-b)$

$\Rightarrow \alpha^2 = (b^2+1)/(1-b)$ ....(ii)

$\alpha = (1+b)/(b-1)$ ...(iii)

Put α in (ii)

$(1+b)^2/(b-1)^2 = (b^2+1)/(1-b)$

$\Rightarrow (1+b)^2(1-b) = (b-1)^2(b^2+1)$

$\Rightarrow -(b-1)(1+b)^2 = (b-1)^2(b^2+1)$

$\Rightarrow -(1+b)^2 = (b-1)(b^2+1)$

$\Rightarrow (-1-2b-b^2) = (b^3 - b^2 + b - 1)$

$\Rightarrow -3b = b^3$

$\Rightarrow b^2 = -3$

Taking square root

$\Rightarrow b = \pm i\sqrt{3}$

Hence option b is the answer.