Question 3: Let $-\pi/6 < \theta < -\pi/12$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals

(a) $2(\sec\theta - \tan\theta)$

(b) $2\sec\theta$

(c) $-2\tan\theta$

(d) 0

Solution:

$x^2 - 2x\sec\theta + 1 = 0$ ...(i)

$\Rightarrow x = [2\sec\theta \pm \sqrt{(4\sec^2\theta - 4)}]/2$

$= \sec\theta \pm \tan\theta$

$x^2 + 2x\tan\theta - 1 = 0$ ...(ii)

$\Rightarrow x = -2\tan\theta \pm \sqrt{(4\tan^2\theta + 4)}/2$

$= -\tan\theta \pm \sec\theta$

Since $-\pi/6 < \theta < -\pi/12$

$\Rightarrow \sec\pi/6 > \sec\theta > \sec\pi/12$

$-\tan\pi/6 < \tan\theta < -\tan\pi/12$

And $\tan\pi/12 < -\tan\theta < \tan\pi/6$

Given $\alpha_1 > \beta_1$

So $\alpha_1 = \sec\theta - \tan\theta$

$\beta_1 = \sec\theta + \tan\theta$

Given $\alpha_2 > \beta_2$

So $\alpha_2 = -\tan\theta + \sec\theta$

$\beta_2 = -\tan\theta - \sec\theta$

So $\alpha_1 + \beta_2 = \sec\theta - \tan\theta + -\tan\theta - \sec\theta$

$= -2\tan\theta$