Question 2: If $x^2 + px - 444p = 0$ has integral roots, where p is a prime number, then the value(s) of p is (are)

(a) 2

(b) 3

(c) 2, 3 and 37

(d) 37

Solution:

Given $x^2 + px - 444p = 0$

Using quadratic formula, $x = -p \pm \sqrt{(p^2 + 4 \times 444p)}/2$

Since p = 2 does not give the integral roots, so D must be a perfect square of an odd integer.

$D = p^2 + 1776p$

$= p(p + 1776)$

Since D is a perfect square, p + 1776 must be a multiple of p.

=> 1776 should be a multiple of p.

We know $1776 = 2^4 \times 3 \times 37$, where p = 2, 3, or 37

Substitute p = 2, 3, or 37

When p = 2, D is not a perfect square.

When p = 3, D is not a perfect square.

When p = 37, D is a perfect square.

Hence option d is the answer.