Que 8: Let $f: R \to (0, \infty)$ and $g: R \to R$ be twice differentiable function such that f'' and g'' are continuous functions on R. Suppose $f'(2) = g(2) = 0, f'' \neq 0$ and $g'(2) \neq 0$ If $\lim_{x \to 2} \frac{f(x)g(x)}{f'(x)g'(x)} = 1$, then **[JEE-(Advanced) 2016]**

$(1) f$ has a local minimum at $x = 2$

$(2) f$ has a local maximum at $x = 2$

$(3) f''(2) > f(2)$

$(4) f(x) - f''(x) = 0$ for at least one $x \in R$

Ans 8:

Using L'Hopital's rule

$$\lim_{x \to 2} \frac{f'(x)g(x) + f(x)g'(x)}{f''(x)g'(x) + f'(x)g''(x)} = 1$$

$$\Rightarrow \frac{f(2)g'(2)}{f''(2)g'(2)} = 1$$

$$\Rightarrow f''(2) = f(2) > 0$$

Option (4) is right and option (3) is wrong

Also $f'(2) = 0$ and $f''(2) > 0$

$\therefore x = 2$ is local minima