Que 7: The least value of $\alpha \in R$ for which $4\alpha x^2 + \frac{1}{x} \geq 1$, for all $x > 0$, is-

[JEE-(Advanced) 2016]

(1) $\frac{1}{64}$

(2) $\frac{1}{32}$

(3) $\frac{1}{27}$

(4) $\frac{1}{25}$

Ans 7:

$f(x) = 4\alpha x^2 + \frac{1}{x}; x > 0$

$f'(x) = 8\alpha x - \frac{1}{x^2} = \frac{8\alpha x^3 - 1}{x^2}$

$f(x)$ attains its minimum at $x = \left(\frac{1}{8\alpha}\right)^{\frac{1}{3}}$

$f\left(\left(\frac{1}{8\alpha}\right)^{\frac{1}{3}}\right) = 1$

$\Rightarrow 4\alpha\left(\frac{1}{8\alpha}\right)^{\frac{2}{3}} + (8\alpha)^{\frac{1}{3}} = 1$

$\Rightarrow 3\alpha^{\frac{1}{3}} = 1 \Rightarrow \alpha = \frac{1}{27}$