Que 6: A cylindrical container is to be made from certain solid material with the following constraints. It has fixed inner volume of $V\ mm^3$, has 2mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2mm and is radius equal to the outer radius of container. If the volume of the material used to make the container is minimum when the inner radius of the container is 10mm, then the value of $\frac{V}{250\pi}$ is _____. **[JEE 2015, 4M, 0M]**

Ans 6:

Let the inner radius of cylindrical container be r then radius of outer cylinder is $(r+2)$

Now, $V = \pi r^2 h$ where h is height of cylinder

Now, let total volume of total material used be T

$$\therefore T = \pi((r+2)^2 - r^2) \times \frac{V}{\pi r^2} + \pi(r+2)^2 2$$

$$\therefore T = V\left(\frac{r+2}{r}\right)^2 + 2\pi(r+2)^2 - V$$

Now, $\frac{dT}{dr} = 2V\left(\frac{r+2}{r}\right) \times \left(-\frac{2}{r^2}\right) + 4\pi(r+2)$

Now at $r = 10mm\ \frac{dT}{dr} = 0$

$$\therefore \quad 0 = (r+2).4\left(\pi - \frac{V}{r^3}\right)$$

$$\Rightarrow \frac{V}{\pi} = 1000 \Rightarrow \frac{V}{250\pi} = 4$$