

Que 2: Let $f(x)$ be polynomial of degree four having extreme values at $x = 1$ and $x =$

2. If $\lim_{x \rightarrow 0} \left[1 + \frac{f(x)}{x^2} \right] = 3$, then $f(2)$ is equal to:

[JEE-MAIN 2015]

(1) 0

(2) 4

(3) - 8

(4) - 4

Ans 2:

Clearly, $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

Now, $\lim_{x \rightarrow 0} \left(1 + \frac{ax^4 + bx^3 + cx^2 + dx + e}{x^2} \right) = 3$

$\therefore d = e = 0$

Now, $\lim_{x \rightarrow 0} (1 + ax^2 + bx + c) = 3$

$\therefore c = 2$

Hence,

$$f(x) = ax^4 + bx^3 + 2x^2$$

$$\begin{aligned} \therefore f'(x) &= 4ax^3 + 3bx^2 + 4x \\ &= x(4ax^2 + 3bx + 4) \end{aligned}$$

Now, $x=1$ and $x=2$ are also solution

$$\therefore 3 = -\frac{3b}{4a} \text{ and } 2 = \frac{4}{4a} \Rightarrow a = \frac{1}{2} \text{ and } b = -2$$

$$\therefore f(x) = \frac{x^4}{2} - 2x^3 + 2x^2$$

$$f(2) = 8 - 16 + 8 = 0$$