Solve for x, $\dfrac{|x+3|+x}{x+2} > 1$

A) $x \in (-5,-2) \cup (-1,\infty)$

B) $x \in (5,2) \cup (-1,\infty)$

C) $x \in (5,2)$

D) $x \in (-1,\infty)$

[a] We have $\dfrac{|x+3|+x}{x+2} > 1$

$\Rightarrow \dfrac{|x+3+x|}{x+2} - 1 > 0$

$\Rightarrow \dfrac{|x+3|-2}{x+2} > 0$ Now two cases arise:

Case I: When $x+3 \geq 0$ i.e., $x \geq -3$.

Then $\dfrac{|x+3|-2}{x+2} > 0 \Rightarrow \dfrac{x+3-2}{x+2} > 0$

$\Rightarrow \dfrac{x+1}{x+2} > 0 \Rightarrow \{(x+1) > 0 \ and \ x+2 > 0\}$

or $\{x+1 < 0 \ and \ x+2 < 0\}$

$\Rightarrow \{x > -1 \ and \ x > -2\}$ or $\{x < -1 \ and \ x < -2\}$

$\Rightarrow x > -1$ or $x < -2 \Rightarrow x \in (-1,\infty)$

or $x \in (-\infty,-2)$

$\Rightarrow x \in (-3,-2) \cup (-1,\infty)$ Since $x \geq -3$

Case II: When $x+3 < 0$, i.e., $x < -3$

$\dfrac{|x+3|-2}{x+2} > 0 \Rightarrow \dfrac{-x-3-2}{x+2} > 0$

$\Rightarrow \dfrac{-(x+5)}{x+2} > 0 \Rightarrow \dfrac{x+5}{x+2} < 0$

$\Rightarrow (x+5 < 0 \ and \ x+2 > 0)$

or $(x+5 > 0 \ and \ x+2 < 0)$

$\Rightarrow \ (x < -5 \ and \ x > -2)$

or $(x > -5 \ and \ x < -2)$ It is not possible.

$\Rightarrow \ x \in (-5, -2)$

Combining (1) and (2), the required solution is $x \in (-5,-2) \cup (-1,\infty)$