

Example 39 Find the absolute maximum and minimum values of a function f given by

$$f(x) = 2x^3 - 15x^2 + 36x + 1 \text{ on the interval } [1, 5].$$

Solution We have

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

or

$$f'(x) = 6x^2 - 30x + 36 = 6(x - 3)(x - 2)$$

Note that $f'(x) = 0$ gives $x = 2$ and $x = 3$.

We shall now evaluate the value of f at these points and at the end points of the interval $[1, 5]$, i.e., at $x = 1$, $x = 2$, $x = 3$ and at $x = 5$. So

$$f(1) = 2(1^3) - 15(1^2) + 36(1) + 1 = 24$$

$$f(2) = 2(2^3) - 15(2^2) + 36(2) + 1 = 29$$

$$f(3) = 2(3^3) - 15(3^2) + 36(3) + 1 = 28$$

$$f(5) = 2(5^3) - 15(5^2) + 36(5) + 1 = 56$$

Thus, we conclude that absolute maximum value of f on $[1, 5]$ is 56, occurring at $x = 5$, and absolute minimum value of f on $[1, 5]$ is 24 which occurs at $x = 1$.