Example 33 Find all the points of local maxima and local minima of the function fgiven by

$$f(x) = 2x^3 - 6x^2 + 6x + 5.$$

Solution We have

$$f(x) = 2x^3 - 6x^2 + 6x + 5.$$

$$f(x) = 2x^3 - 6x^2 + 6x + 5$$

$$\begin{cases} f'(x) = 6x^2 - 12x + 6 = 6(x - 1)^2 \\ f''(x) = 12(x - 1) \end{cases}$$

or

Now f'(x) = 0 gives x = 1. Also f''(1) = 0. Therefore, the second derivative test fails in this case. So, we shall go back to the first derivative test.

We have already seen (Example 30) that, using first derivative test, x = 1 is neither a point of local maxima nor a point of local minima and so it is a point of inflexion.