Q14 : Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

Answer :

The two numbers are x and y such that $x + y = 60$.

$\Rightarrow y = 60 - x$

Let $f(x) = xy^3$.

$$\Rightarrow f(x) = x(60-x)^3$$

$$\therefore f'(x) = (60-x)^3 - 3x(60-x)^2$$

$$= (60-x)^2 [60 - x - 3x]$$

$$= (60-x)^2 (60 - 4x)$$

And, $f''(x) = -2(60-x)(60-4x) - 4(60-x)^2$

$$= -2(60-x)\left[60 - 4x + 2(60-x)\right]$$

$$= -2(60-x)(180 - 6x)$$

$$= -12(60-x)(30-x)$$

Now, $f'(x) = 0 \Rightarrow x = 60$ or $x = 15$

When $x = 60$, $f''(x) = 0$.

When $x = 15$, $f''(x) = -12(60-15)(30-15) = -12 \times 45 \times 15 < 0$.

$\therefore$By second derivative test, $x = 15$ is a point of local maxima of f. Thus, function xy^3 is maximum when $x = 15$ and $y = 60 - 15 = 45$.

Hence, the required numbers are 15 and 45.