Q9 : What is the maximum value of the function sin x + cos x?

Answer :

Let $f(x) = \sin x + \cos x$.

$\therefore f'(x) = \cos x - \sin x$

$f'(x) = 0 \Rightarrow \sin x = \cos x \Rightarrow \tan x = 1 \Rightarrow x = \dfrac{\pi}{4}, \dfrac{5\pi}{4} ....$

$f''(x) = -\sin x - \cos x = -(\sin x + \cos x)$

Now, $f''(x)$ will be negative when (sin x + cos x) is positive i.e., when sin x and cos x are both positive. Also, we know that sin x and cos x both are positive in the first quadrant. Then, $f''(x)$ will be negative when $x \in \left(0, \dfrac{\pi}{2}\right)$.

Thus, we consider $x = \dfrac{\pi}{4}$.

$f''\left(\dfrac{\pi}{4}\right) = -\left(\sin \dfrac{\pi}{4} + \cos \dfrac{\pi}{4}\right) = -\left(\dfrac{2}{\sqrt{2}}\right) = -\sqrt{2} < 0$

$\therefore$ By second derivative test, f will be the maximum at $x = \dfrac{\pi}{4}$ and the maximum value of f is

$f\left(\dfrac{\pi}{4}\right) = \sin \dfrac{\pi}{4} + \cos \dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$.