Example 32 Find local maximum and local minimum values of the function f given by

$$f(x) = 3x^4 + 4x^3 - 12x^2 + 12$$

Solution We have

$$f(x) = 3x^4 + 4x^3 - 12x^2 + 12$$

or $$f'(x) = 12x^3 + 12x^2 - 24x = 12x(x-1)(x+2)$$

or $$f'(x) = 0 \text{ at } x = 0,\ x = 1 \text{ and } x = -2.$$

Now $$f''(x) = 36x^2 + 24x - 24 = 12(3x^2 + 2x - 2)$$

or

$$\begin{cases} f''(0) = -24 < 0 \\ f''(1) = 36 > 0 \\ f''(-2) = 72 > 0 \end{cases}$$

Therefore, by second derivative test, $x = 0$ is a point of local maxima and local maximum value of f at $x = 0$ is $f(0) = 12$ while $x = 1$ and $x = -2$ are the points of local minima and local minimum values of f at $x = -1$ and -2 are $f(1) = 7$ and $f(-2) = -20$, respectively.