Q7 : Find both the maximum value and the minimum value of

$3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval [0, 3]

Answer :

Let $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$.

$$\begin{aligned}\therefore f'(x) &= 12x^3 - 24x^2 + 24x - 48 \\ &= 12(x^3 - 2x^2 + 2x - 4) \\ &= 12[x^2(x-2) + 2(x-2)] \\ &= 12(x-2)(x^2+2)\end{aligned}$$

Now, $f'(x) = 0$ gives $x = 2$ or $x^2 + 2 = 0$ for which there are no real roots.

Therefore, we consider only $x = 2 \in [0, 3]$.

Now, we evaluate the value of f at critical point $x = 2$ and at the end points of the interval [0, 3].

$$\begin{aligned}f(2) &= 3(16) - 8(8) + 12(4) - 48(2) + 25 \\ &= 48 - 64 + 48 - 96 + 25 \\ &= -39 \\ f(0) &= 3(0) - 8(0) + 12(0) - 48(0) + 25 \\ &= 25 \\ f(3) &= 3(81) - 8(27) + 12(9) - 48(3) + 25 \\ &= 243 - 216 + 108 - 144 + 25 = 16\end{aligned}$$

Hence, we can conclude that the absolute maximum value of f on [0, 3] is 25 occurring at $x = 0$ and the absolute minimum value of f at [0, 3] is - 39 occurring at $x = 2$.