Example 9 Prove that in a Δ ABC, $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$, where a, b, c represent the magnitudes of the sides opposite to vertices A, B, C, respectively.

Solution Let the three sides of the triangle BC, CA and AB be represented by $\vec{a}, \vec{b}$ and $\vec{c}$, respectively [Fig. 10.2].

We have $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. i.e., $\vec{a}+\vec{b}=-\vec{c}$

which pre cross multiplying by $\vec{a}$, and

post cross multiplying by $\vec{b}$, gives

$$\vec{a}\times\vec{b}=\vec{c}\times\vec{a}$$

and $$\vec{a}\times\vec{b}=\vec{b}\times\vec{c}$$

respectively. Therefore,

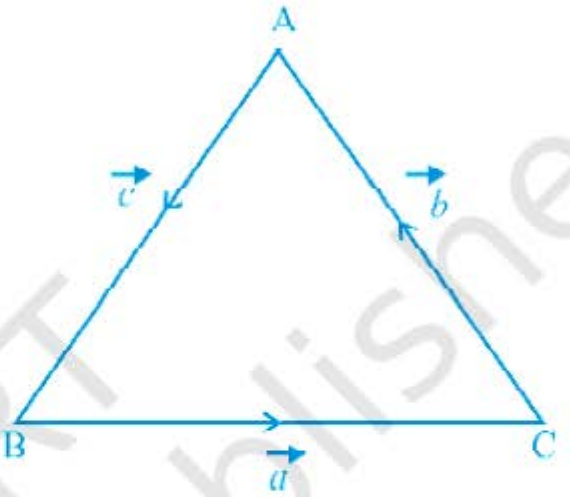

Fig. 10.2

$$\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$$

$$\Rightarrow \quad |\vec{a}\times\vec{b}|=|\vec{b}\times\vec{c}|=|\vec{c}\times\vec{a}|$$

$$\Rightarrow \quad |\vec{a}||\vec{b}|\sin(\pi-C)=|\vec{b}||\vec{c}|\sin(\pi-A)=|\vec{c}||\vec{a}|\sin(\pi-B)$$

$$\Rightarrow \quad ab \sin C = bc \sin A = ca \sin B$$

Dividing by abc, we get

$$\frac{\sin C}{c}=\frac{\sin A}{a}=\frac{\sin B}{b} \text{ i.e. } \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$