Example 7 Find all vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane of $\hat{i}+2\hat{j}+\hat{k}$ and $-\hat{i}+3\hat{j}+4\hat{k}$.

Solution Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$ and $\vec{b}=-\hat{i}+3\hat{j}+4\hat{k}$. Then

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 1\\ -1 & 3 & 4\end{vmatrix}=\hat{i}(8-3)-\hat{j}(4+1)+\hat{k}(3+2) \quad =5\hat{i}-5\hat{j}+5\hat{k}$$

$$\Rightarrow \qquad |\vec{a}\times\vec{b}|=\sqrt{(5)^2+(-5)^2+(5)^2}=\sqrt{3(5)^2}=5\sqrt{3}.$$