If the vectors $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ from the sides BC, CA and AB respectively of a ΔABC, then

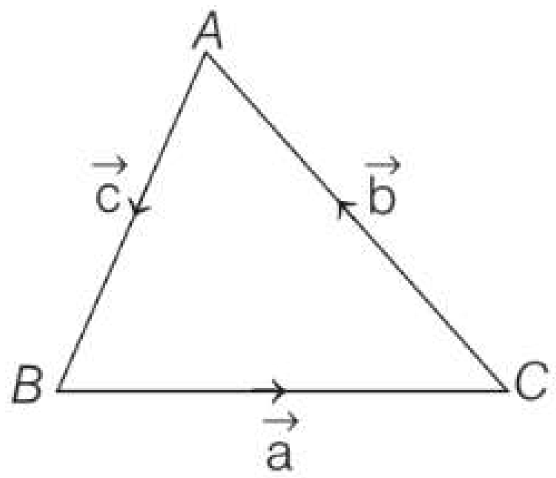
(a) $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}} = 0$ **(2000, 2M)**

(b) $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{b}} \times \vec{\mathbf{c}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}}$

(c) $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = \vec{\mathbf{c}} \cdot \vec{\mathbf{a}}$

(d) $\vec{\mathbf{a}} \times \vec{\mathbf{b}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} = \vec{\mathbf{0}}$

By triangle law, $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$



Taking cross product by $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, $\vec{\mathbf{c}}$ respectively,

$$\vec{\mathbf{a}} \times (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}) = \vec{\mathbf{a}} \times \vec{\mathbf{0}} = \vec{\mathbf{0}}$$

$$\Rightarrow \quad \vec{\mathbf{a}} \times \vec{\mathbf{a}} + \vec{\mathbf{a}} \times \vec{\mathbf{b}} + \vec{\mathbf{a}} \times \vec{\mathbf{c}} = \vec{\mathbf{a}}$$

$$\Rightarrow \quad \vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}} \qquad [\because \vec{\mathbf{a}} \times \vec{\mathbf{a}} = \vec{0}]$$

Similarly, $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{b}} \times \vec{\mathbf{a}}$

$$\therefore \quad \vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{b}} \times \vec{\mathbf{c}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}}$$