Let $\vec{\alpha} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\vec{\beta} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to **(2019 Main, 9 April I)**

(a) $\frac{1}{2}(3\hat{\mathbf{i}} - 9\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$
(b) $\frac{1}{2}(-3\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$
(c) $-3\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$
(d) $3\hat{\mathbf{i}} - 9\hat{\mathbf{j}} - 5\hat{\mathbf{k}})$

Given vectors $\vec{\alpha} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\vec{\beta} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$ such that $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to α

So, $\vec{\beta}_1 = \lambda\alpha = \lambda(3\hat{\mathbf{i}} + \hat{\mathbf{j}})$

Now, $\vec{\beta}_2 = \vec{\beta}_1 - \vec{\beta} = \lambda(3\hat{\mathbf{i}} + \hat{\mathbf{j}}) - (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

$= (3\lambda - 2)\hat{\mathbf{i}} + (\lambda + 1)\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

$\because \vec{\beta}_2$ is perpendicular to α, so $\vec{\beta}_2 \cdot \alpha = 0$

[since if non-zero vectors **a** and **b** are perpendicular to each other, then $\mathbf{a} \cdot \mathbf{b} = 0$]

$\therefore \quad (3\lambda - 2)(3) + (\lambda + 1)(1) = 0$

$\Rightarrow \quad 9\lambda - 6 + \lambda + 1 = 0$

$\Rightarrow \quad 10\lambda = 5 \Rightarrow \lambda = \frac{1}{2}$

So, $\vec{\beta}_1 = \frac{3}{2}\hat{\mathbf{i}} + \frac{1}{2}\hat{\mathbf{j}}$

and $\vec{\beta}_2 = \left(\frac{3}{2} - 2\right)\hat{\mathbf{i}} + \left(\frac{1}{2} + 1\right)\hat{\mathbf{j}} - 3\hat{\mathbf{k}} = -\frac{1}{2}\hat{\mathbf{i}} + \frac{3}{2}\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

$$\therefore \vec{\beta}_1 \times \vec{\beta}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \frac{3}{2} & \frac{1}{2} & 0 \\ \frac{-1}{2} & \frac{3}{2} & -3 \end{vmatrix} = \hat{\mathbf{i}}\left(-\frac{3}{2} - 0\right) - \hat{\mathbf{j}}\left(-\frac{9}{2} - 0\right) + \hat{\mathbf{k}}\left(\frac{9}{4} + \frac{1}{4}\right)$$

$$= -\frac{3}{2}\hat{\mathbf{i}} + \frac{9}{2}\hat{\mathbf{j}} + \frac{5}{2}\hat{\mathbf{k}} = \frac{1}{2}(-3\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$$