Let $\mathbf{a}=3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+x\hat{\mathbf{k}}$ and $\mathbf{b}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$, for some real x. Then $|\mathbf{a}\times\mathbf{b}|=r$ is possible if

[JEE Main 2019, 8 April Shift-II]

(a) $0<r\le\sqrt{\frac{3}{2}}$ (b) $\sqrt{\frac{3}{2}}<r\le 3\sqrt{\frac{3}{2}}$

(c) $3\sqrt{\frac{3}{2}}<r<5\sqrt{\frac{3}{2}}$ (d) $r\ge 5\sqrt{\frac{3}{2}}$

Exp. (d)

Given vectors are $\mathbf{a}=3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+x\hat{\mathbf{k}}$

and $\mathbf{b}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$$\therefore \quad \mathbf{a}\times\mathbf{b}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 3 & 2 & x\\ 1 & -1 & 1\end{vmatrix}$$

$$=\hat{\mathbf{i}}(2+x)-\hat{\mathbf{j}}(3-x)+\hat{\mathbf{k}}(-3-2)$$

$$=(x+2)\hat{\mathbf{i}}+(x-3)\hat{\mathbf{j}}-5\hat{\mathbf{k}}$$

$$\Rightarrow \quad |\mathbf{a}\times\mathbf{b}|=\sqrt{(x+2)^2+(x-3)^2+25}$$

$$=\sqrt{2x^2-2x+4+9+25}$$

$$=\sqrt{2\left(x^2-x+\frac{1}{4}\right)-\frac{1}{2}+38}$$

$$=\sqrt{2\left(x-\frac{1}{2}\right)^2+\frac{75}{2}}$$

So, $|\mathbf{a}\times\mathbf{b}|\ge\sqrt{\frac{75}{2}}$ [at $x=\frac{1}{2}$, $|\mathbf{a}\times\mathbf{b}|$ is minimum]

$$\Rightarrow \quad r\ge 5\sqrt{\frac{3}{2}}$$