Assuming that the moon is a sphere of the same mean density as that of the earth and one quarter of its radius, the length of a seconds pendulum on the moon (its length on the earth's surface is 99.2 cm) is:

A. 24.8 cm

B. 49.6 cm

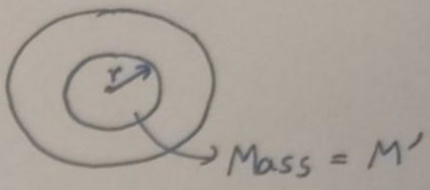
C. 99.2

D. $\frac{99.2}{\sqrt{2}}$ cm

Solution

① $\rho \propto \frac{1}{r}$ (Given)

Let, $\rho = \frac{k}{r}$



$$\frac{dM}{dV} = \frac{k}{r}$$

$$dM = \frac{k}{r} \times 4\pi r^2 dr$$

$$\int_0^{M'} dM = 4\pi k \int_0^r r\,dr$$

$$M' = 4\pi k \left[\frac{r^2}{2}\right]_0^r$$

$$M' = 2\pi k r \quad — ①$$

$$g' = \frac{GM'}{r^2} = \frac{G \times 2\pi k r^2}{r^2} = \underline{G 2\pi k}$$

(at r)

g' is independent of r.

Ans ⇒ (D)

② $R_{MOON} = \frac{R_e}{4}$, ⇒ $V_{MOON} = \frac{V_e}{64}$

Thus, $M_{MOON} = \frac{M_e}{64}$ (Density is same)

$$g_{MOON} = \frac{G M_{MOON}}{R_{MOON}^2} = \frac{g}{4}$$

We know that ⇒ $T = 2\pi\sqrt{\frac{l}{g}}$

$$l = gT^2/4\pi^2$$

As $g \propto l$, length on moon will be $(1/4)^{th}$ of length on earth. ⇒ $l = 99.2/4 = 24.8$ cm ⇒ (A)