$$g(h) = \frac{F(h)}{m} = \frac{GM_E}{(R_E + h)^2}. \tag{8.14}$$

This is clearly less than the value of g on the surface of earth : $g = \frac{GM_E}{R_E^2}$. For $h << R_E$, we can expand the RHS of Eq. (8.14) :

$$g(h) = \frac{GM_E}{R_E^2(1 + h/R_E)^2} = g(1 + h/R_E)^{-2}$$

For $\frac{h}{R_E} << 1$, using binomial expression,

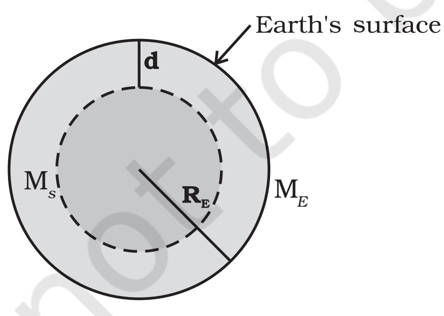
$$g(h) \cong g\left(1 - \frac{2h}{R_E}\right). \tag{8.15}$$

Equation (8.15) thus tells us that for small heights h above the value of g decreases by a factor $(1 - 2h/R_E)$.

Now, consider a point mass m at a depth d below the surface of the earth (Fig. 8.8(b)), so that its distance from the centre of the earth is $(R_E - d)$ as shown in the figure. The earth can be thought of as being composed of a smaller sphere of radius $(R_E - d)$ and a spherical shell of thickness d. The force on m due to the outer shell of thickness d is zero because the result quoted in the previous section. As far as the smaller sphere of radius $(R_E - d)$ is concerned, the point mass is outside it and hence according to the result quoted earlier, the force due to this smaller sphere is just as if the entire mass of the smaller sphere is concentrated at the centre. If M_s is the mass of the smaller sphere, then,

$$M_s / M_E = (R_E - d)^3 / R_E^3 \tag{8.16}$$

Since mass of a sphere is proportional to be cube of its radius.



(b)

Fig. 8.8 (b) *g at a depth d. In this case only the smaller sphere of radius (R_E–d) contributes to g.*

Thus the force on the point mass is

$$F(d) = G\, M_s\, m\, /\, (R_E - d)^2 \tag{8.17}$$

Substituting for M_s from above , we get

$$F(d) = G\, M_E\, m\, (R_E - d)\, /\, R_E^3 \tag{8.18}$$

and hence the acceleration due to gravity at a depth d,

$$g(d) = \frac{F(d)}{m} \text{ is}$$

$$g(d) = \frac{F(d)}{m} = \frac{GM_E}{R_E^3}(R_E - d)$$

$$= g\frac{R_E - d}{R_E} = g(1 - d/R_E) \tag{8.19}$$

Thus, as we go down below earth's surface, the acceleration due gravity decreases by a factor $(1 - d/R_E)$. The remarkable thing about acceleration due to earth's gravity is that it is maximum on its surface decreasing whether you go up or down.

8.7 GRAVITATIONAL POTENTIAL ENERGY

We had discussed earlier the notion of potential energy as being the energy stored in the body at its given position. If the position of the particle changes on account of forces acting on it, then the change in its potential energy is just the amount of work done on the body by the force. As we had discussed earlier, forces for which the work done is independent of the path are the conservative forces.

The force of gravity is a conservative force and we can calculate the potential energy of a body arising out of this force, called the gravitational potential energy. Consider points close to the surface of earth, at distances from the surface much smaller than the radius of the earth. In such cases, the force of gravity is practically a constant equal to mg, directed towards the centre of the earth. If we consider a point at a height h_1 from the surface of the earth and another point vertically above it at a height h_2 from the surface, the work done in lifting the particle of mass m from the first to the second position is denoted by W_{12}

$$W_{12} = \text{Force} \times \text{displacement}$$
$$= mg\,(h_2 - h_1) \tag{8.20}$$