Let $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \sqrt{2}\,\hat{\mathbf{k}}, \mathbf{b} = b_1\,\hat{\mathbf{i}} + b_2\,\hat{\mathbf{j}} + \sqrt{2}\,\hat{\mathbf{k}}$ and $\mathbf{c} = 5\,\hat{\mathbf{i}} + \hat{\mathbf{j}} + \sqrt{2}\,\hat{\mathbf{k}}$ be three vectors such that the projection vector of $\mathbf{b}$ on $\mathbf{a}$ is $\mathbf{a}$. If $\mathbf{a} + \mathbf{b}$ is perpendicular to $\mathbf{c}$, then $|\mathbf{b}|$ is equal to

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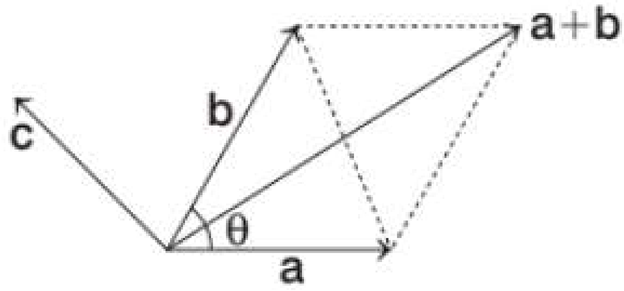
(a) 6 (b) 4 (c) $\sqrt{22}$ (d) $\sqrt{32}$

According to given information, we have the following figure.



Clearly, projection of b on a $= \frac{b \cdot a}{|a|}$

$$= \frac{(b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k})\,(\hat{i} + \hat{j} + \sqrt{2}\hat{k})}{\sqrt{1^2 + 1^2 + (\sqrt{2})^2}}$$

$$= \frac{b_1 + b_2 + 2}{\sqrt{4}} = \frac{b_1 + b_2 + 2}{2}$$

But projection of b on a $= |a|$

$$\therefore \quad \frac{b_1 + b_2 + 2}{2} = \sqrt{1^2 + 1^2 + (\sqrt{2})^2}$$

$$\Rightarrow \frac{b_1 + b_2 + 2}{2} = 2 \Rightarrow b_1 + b_2 = 2 \qquad \text{...(i)}$$

Now, $a + b = (\hat{i} + \hat{j} + \sqrt{2}\hat{k}) + (b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k})$

$$= (b_1 + 1)\hat{i} + (b_2 + 1)\hat{j} + 2\sqrt{2}\hat{k}$$

$\because (a + b) \perp c$, therefore $(a + b) \cdot c = 0$

$$\Rightarrow \quad \{(b_1 + 1)\hat{i} + (b_2 + 1)\hat{j} + 2\sqrt{2}\hat{k}\}\,(5\hat{i} + \hat{j} + \sqrt{2}\hat{k}) = 0$$

$$\Rightarrow \quad 5(b_1 + 1) + 1(b_2 + 1) + 2\sqrt{2}(\sqrt{2}) = 0$$

$$\Rightarrow \quad 5b_1 + b_2 = -10 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), $b_1 = -3$ and $b_2 = 5$

$$\Rightarrow \quad b = -3\hat{i} + 5\hat{j} + \sqrt{2}\hat{k}$$

$$\Rightarrow |b| = \sqrt{(-3)^2 + (5)^2 + (\sqrt{2})^2} = \sqrt{36} = 6$$