Two adjacent sides of a parallelogram $ABCD$ are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by **(2010)**

(a) $\frac{8}{9}$ (b) $\frac{\sqrt{17}}{9}$ (c) $\frac{1}{9}$ (d) $\frac{4\sqrt{5}}{9}$

$$\overrightarrow{\mathbf{AB}} = 2\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 11\hat{\mathbf{k}}, \quad \overrightarrow{\mathbf{AD}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$

Angle 'θ' between $\overrightarrow{\mathbf{AB}}$ and $\overrightarrow{\mathbf{AD}}$ is

$$\cos(\theta) = \left| \frac{\overrightarrow{\mathbf{AB}} \cdot \overrightarrow{\mathbf{AD}}}{|\overrightarrow{\mathbf{AB}}||\overrightarrow{\mathbf{AD}}|} \right| = \left| \frac{-2+20+22}{(15)(3)} \right| = \frac{8}{9}$$

$$\Rightarrow \qquad \sin(\theta) = \frac{\sqrt{17}}{9}$$

Since, $\alpha + \theta = 90^\circ$

$$\therefore \qquad \cos(\alpha) = \cos(90^\circ - \theta) = \sin(\theta) = \frac{\sqrt{17}}{9}$$