8.14 Supposing Newton's law of gravitation for gravitation forces $\mathbf{F}_1$ and $\mathbf{F}_2$ between two masses m_1 and m_2 at positions $\mathbf{r}_1$ and $\mathbf{r}_2$ read

$\mathbf{F}_1 = -\mathbf{F}_2 = -\frac{\mathbf{r}_{12}}{r_{12}^3} GM_0^2 \left(\frac{m_1 m_2}{M_0^2}\right)^n$ where M_0 is a constant of dimension of mass, $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$ and n is a number. In such a case,

(a) the acceleration due to gravity on earth will be different for different objects.

(b) none of the three laws of Kepler will be valid.

(c) only the third law will become invalid.

(d) for n negative, an object lighter than water will sink in water.

Solution

14) $\vec{F_1} = -\vec{F_2} = -\frac{\vec{r}_{12}}{r_{12}^2} G M_0^2 \left(\frac{m_1 m_2}{M_0^2}\right)^n$

Take, $m_1 = M_e$, $m_2 = m$ (mass of object),

$r_{12} = R_e$

Therefore, $F = \left(\frac{G M_0^{(2-2n)} M_e^n}{R_e^2}\right) m^n = K m^n$

K is some constant.

$F = mg = K m^n$

$$\boxed{g = K m^{n-1}}$$

So, g is not independent of m. (a)
And g is dependent on position vector.
As, g is not constant, hence constant of proportionality will not be constant in Kepler's 3rd Law. So, it will not be valid. (c)

As the force is central in nature, Kepler's first & second law is still valid.

When $n < 0$, $F = K/m^n$.

This implies that lighter bodies will experience a greater force than the heavier bodies.
Hence, objects lighter than water will sink. (d)

Ans ⇒ (a, c, d)