Reflection of sound in an open pipe

When a high pressure pulse of air traveling down an open pipe reaches the other end, its momentum drags the air out into the open, where pressure falls rapidly to the atmospheric pressure. As a result the air following after it in the tube is pushed out. The low pressure at the end of the tube draws air from further up the tube. The air gets drawn towards the open end forcing the low pressure region to move upwards. As a result a pulse of *high* pressure air travelling *down* the tube turns into a pulse of *low* pressure air travelling *up* the tube. We say a pressure wave has been reflected at the open end with a change in phase of 180^0. Standing waves in an open pipe organ like the flute is a result of this phenomenon.

Compare this with what happens when a pulse of high pressure air arrives at a closed end: it collides and as a result pushes the air back in the opposite direction. Here, we say that the pressure wave is reflected, with no change in phase.

frequencies is a wave with nearly same angular frequency but its amplitude is not constant. Thus the intensity of resultant sound varies with an angular frequency $\omega_{beat} = 2\,\omega_b = \omega_1 - \omega_2$. Now using the relation,

$$\omega = 2\pi\nu$$

the beat frequency, ν_{beat}, is given by

$$\nu_{beat} = \nu_1 - \nu_2 \qquad (15.48)$$

Thus we hear a waxing and waning of sound with a frequency equal to the difference between the frequencies of the superposing waves. The time-displacement graphs of two waves of frequency 11 Hz and 9 Hz is shown in Figs. 15.16(a) and 15.16(b). The result of their 'superposition' is shown in Fig. 15.16(c).

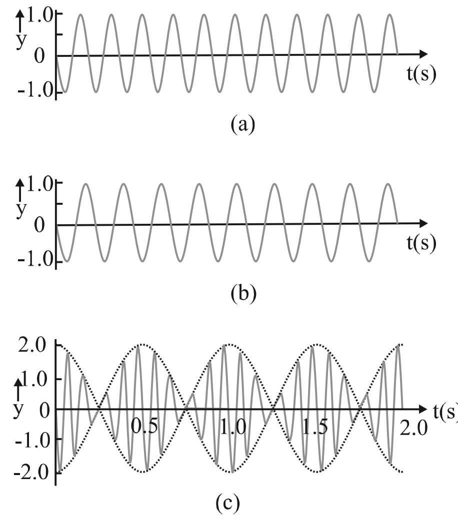

Fig. 15.16 *(a) Plot of a harmonic wave of frequency 11 Hz. (b) Plot of a harmonic wave of frequency 9 Hz. (c) Superposition of (a) and (b), showing clearly the beats in the slow (2 Hz) of the total disturbance.*

Musicians use the beat phenomenon in tuning their instruments. If an instrument is sounded against a standard frequency and tuned until the beat disappears, then the instrument is in tune with that standard.

▶ ***Example 15.6*** Two sitar strings A and B playing the note '*Dha*' are slightly out of tune and produce beats of frequency 5 Hz. The tension of the string B is slightly increased and the beat frequency is found to decrease to 3 Hz. What is the original frequency of B if the frequency of A is 427 Hz ?

Answer Increase in the tension of a string increases its frequency. If the original frequency of B (ν_B) were greater than that of A (ν_A), further increase in ν_B should have resulted in an increase in the beat frequency. But the beat frequency is found to decrease. This shows that $\nu_B < \nu_A$. Since $\nu_A - \nu_B = 5$ Hz, and $\nu_A = 427$ Hz, we get $\nu_B = 422$ Hz. ◀

15.8 DOPPLER EFFECT

It is an everyday experience that the pitch (or frequency) of the whistle of a fast moving train decreases as it recedes away. When we approach a stationary source of sound with high speed, the pitch of the sound heard appears to be higher than that of the source. As the observer recedes away from the source, the observed **pitch** (or frequency) becomes lower than that of the source. This motion-related frequency change is called **Doppler effect**. The Austrian physicist Johann Christian Doppler first proposed the effect in 1842. Buys Ballot in Holland tested it experimentally in 1845. Doppler effect is a wave phenomenon, it holds not only for sound waves but also for electromagnetic waves. However, here we shall consider only sound waves.

We shall analyse changes in frequency under three different situations: (1) observer is stationary but the source is moving, (2) observer is moving but the source is stationary, and (3) both the observer and the source are moving. The situations (1) and (2) differ from each other because of the absence or presence of relative motion between the observer and the medium. Most waves require a medium for their propagation; however, electromagnetic waves do not require any medium for propagation. If there is no medium present, the Doppler shifts are same irrespective of whether the source moves or the observer moves, since there is no way of distinction between the two situations.

15.8.1 Source Moving ; Observer Stationary

Let us choose the convention to take the direction from the observer to the source as the positive direction of velocity. Consider a source S moving with velocity υ_s and an observer who is stationary in a frame in which the medium is also at rest. Let the speed of a wave of angular frequency ω and period T_o, both measured by an observer at rest with respect to the medium, be υ. We assume that the observer has a detector that counts every time a wave crest reaches it. As shown in Fig. 15.17, at time $t = 0$ the source is at point S_1, located at a distance L from the observer, and emits a crest. This reaches the observer at time $t_1 = L/\upsilon$. At time $t = T_o$ the source has moved a distance $\upsilon_s T_o$ and is at point S_2, located at a distance $(L + \upsilon_s T_o)$ from the observer. At S_2, the source emits a second crest. This reaches the observer at

$$t_2 = T_0 + \frac{(L + \upsilon_s T_0)}{\upsilon}$$

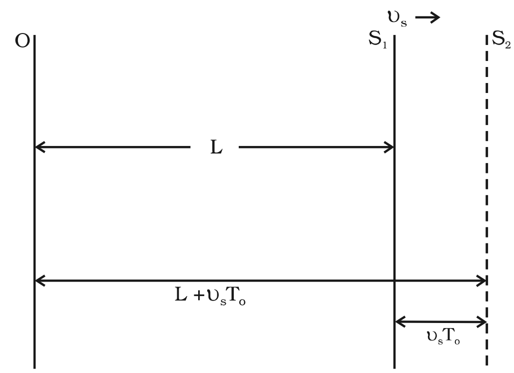

Fig. 15.17 *A source moving with velocity υ_s emits a wave crest at the point S_1. It emits the next wave crest at S_2 after moving a distance $\upsilon_s T_o$.*

At time $n\,T_o$, the source emits its $(n+1)^{th}$ crest and this reaches the observer at time

$$t_{n+1} = n\,T_0 + \frac{L + n\upsilon_s T_0}{\upsilon}$$

Hence, in a time interval

$$\left[nT_0 + \frac{L + n\upsilon_s T_0}{\upsilon} - \frac{L}{\upsilon} \right]$$

the observer's detector counts n crests and the observer records the period of the wave as T given by

$$T = \left[nT_0 + \frac{L + n\upsilon_s T_0}{\upsilon} - \frac{L}{\upsilon} \right] / n$$

$$= T_0 + \frac{\upsilon_s T_0}{\upsilon}$$

$$= T_0 \left(1 + \frac{\upsilon_s}{\upsilon} \right) \quad (15.49)$$

Equation (15.49) may be rewritten in terms of the frequency ν_o that would be measured if the source and observer were stationary, and the frequency ν observed when the source is moving, as

$$\nu = \nu_0 \left(1 + \frac{\upsilon_s}{\upsilon} \right)^{-1} \quad (15.50)$$

If υ_s is small compared with the wave speed υ, taking binomial expansion to terms in first order in υ_s/υ and neglecting higher power, Eq. (15.50) may be approximated, giving

$$\nu = \nu_0 \left(1 - \frac{\upsilon_s}{\upsilon} \right) \quad (15.51)$$

For a source approaching the observer, we replace υ_s by $-\upsilon_s$ to get

$$\nu = \nu_0 \left(1 + \frac{v_s}{v}\right) \qquad (15.52)$$

The observer thus measures a lower frequency when the source recedes from him than he does when it is at rest. He measures a higher frequency when the source approaches him.

15.8.2 Observer Moving; Source Stationary

Now to derive the Doppler shift when the observer is moving with velocity v_o towards the source and the source is at rest, we have to proceed in a different manner. We work in the reference frame of the moving observer. In this reference frame the source and medium are approaching at speed v_o and the speed with which the wave approaches is $v_o + v$. Following a similar procedure as in the previous case, we find that the time interval between the arrival of the first and the $(n+1)$ th crests is

$$t_{n+1} - t_1 = n\ T_0 - \frac{n v_0 T_0}{v_0 + v}$$

The observer thus, measures the period of the wave to be

$$= T_0 \left(1 - \frac{v_0}{v_0 + v}\right)$$

$$T_0 \quad 1 \quad \frac{v_0}{v} \quad ^{-1}$$

giving

$$\nu = {}_0 \left(1 + \frac{v_0}{v}\right) \qquad (15.53)$$

If $\frac{v_0}{v}$ is small, the Doppler shift is almost same whether it is the observer or the source moving since Eq. (15.53) and the approximate relation Eq. (15.51) are the same.

15.8.3 Both Source and Observer Moving

We will now derive a general expression for Doppler shift when both the source and the observer are moving. As before, let us take the direction from the observer to the source as the positive direction. Let the source and the observer be moving with velocities v_s and v_o respectively as shown in Fig.15.18. Suppose at time $t = 0$, the observer is at O_1 and the source is at S_1, O_1 being to the left of S_1. The source emits a wave of velocity v, of frequency υ and period T_0 all measured by an observer at rest with respect to the medium. Let L be the distance between O_1 and S_1 at $t = 0$, when the source emits the first crest. Now, since the observer is moving, the velocity of the wave relative to the observer is $v + v_0$. Therfore the first crest reaches the observer at time $t_1 = L/(v+v_0)$. At time $t = T_0$, both the observer and the source have moved to their new positions O_2 and S_2 respectively. The new distance between the observer and the source, $O_2 S_2$, would be $L+(v_s- v_0)\ T_0]$. At S_2, the source emits a second crest. This reaches the observer at time.

$$t_2 = T_o + [L + (v_s - v_o)T_o)]\ /(v + v_o)$$

At time nT_o the source emits its $(n+1)$ th crest and this reaches the observer at time

$$t_{n+1} = nT_o + [L + n\,(v_s - v_o)T_o)]\ /(v + v_o)$$

Hence, in a time interval $t_{n+1} - t_1$, i.e.,

$$nT_o + [L + n\,(v_s - v_o)T_o)]\ /(v + v_o) - L\,/(v + v_o),$$

the observer counts n crests and the observer records the period of the wave as equal to T given by

Application of Doppler effect

The change in frequency caused by a moving object due to Doppler effect is used to measure their velocities in diverse areas such as military, medical science, astrophysics, etc. It is also used by police to check over-speeding of vehicles.

A sound wave or electromagnetic wave of known frequency is sent towards a moving object. Some part of the wave is reflected from the object and its frequency is detected by the monitoring station. This change in frequency is called **Doppler shift**.

It is used at airports to guide aircraft, and in the military to detect enemy aircraft. Astrophysicists use it to measure the velocities of stars.

Doctors use it to study heart beats and blood flow in different part of the body. Here they use ulltrasonic waves, and in common practice, it is called **sonography**. Ultrasonic waves enter the body of the person, some of them are reflected back, and give information about motion of blood and pulsation of heart valves, as well as pulsation of the heart of the foetus. In the case of heart, the picture generated is called **echocardiogram**.

$$T = T_0\left(1+\frac{\upsilon_s - \upsilon_o}{\upsilon + \upsilon_0}\right) = T_0\left(\frac{\upsilon+\upsilon_s}{\upsilon+\upsilon_0}\right) \quad (15.54)$$

The frequency ν observed by the observer is given by

$$\nu = \nu_0\left(\frac{\upsilon+\upsilon_0}{\upsilon+\upsilon_s}\right) \quad (15.55)$$

Consider a passenger sitting in a train moving on a straight track. Suppose she hears a whistle sounded by the driver of the train. What frequency will she measure or hear? Here both the observer and the source are moving with the same velocity, so there will be no shift in frequency and the passanger will note the natural frequency. But an observer outside who is stationary with respect to the track will note a higher frequency if the train is approaching him and a lower frequency when it recedes from him.

Note that we have defined the direction from the observer to the source as the positive

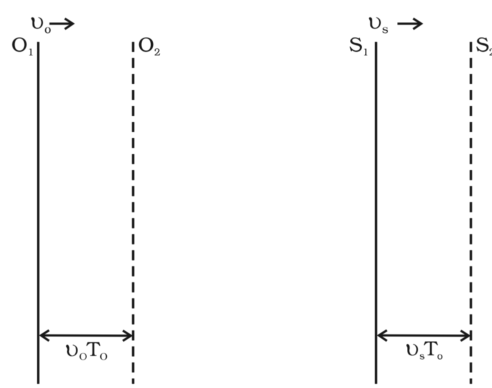

Fig. 15.18 *The observer* O *and the source* S, *both moving respectively with velocities* υ_0 *and* υ_s. *They are at position* O_1 *and* S_1 *at time* $t = 0$, *when the source emits the first crest of a sound, whose velocity is* υ *with respect to the medium. After one period,* $t = T_0$, *they have moved to* O_2 *and* S_2, *respectively through distances* $\upsilon_0 T_0$ *and* $\upsilon_s T_0$, *when the source emits the next crest.*

direction. Therefore, if the observer is moving towards the source, υ_0 has a positive (numerical) value whereas if O is moving away from S, υ_0 has a negative value. On the other hand, if S is moving away from O, υ_s has a positive value whereas if it is moving towards O, υ_s has a negative value. The sound emitted by the source travels in all directions. It is that part of sound coming towards the observer which the observer receives and detects. Therefore the relative velocity of sound with respect to the observer is $\upsilon + \upsilon_0$ in all cases.

Example 15.7 A rocket is moving at a speed of 200 m s^{-1} towards a stationary target. While moving, it emits a wave of frequency 1000 Hz. Some of the sound reaching the target gets reflected back to the rocket as an echo. Calculate (1) the frequency of the sound as detected by the target and (2) the frequency of the echo as detected by the rocket.

Answer (1) The observer is at rest and the source is moving with a speed of 200 m s^{-1}. Since this is comparable with the velocity of sound, 330 m s^{-1}, we must use Eq. (15.50) and not the approximate Eq. (15.51). Since the source is approaching a stationary target, $\upsilon_o = 0$, and υ_s must be replaced by $-\upsilon_s$. Thus, we have

$$\nu = \nu_0\left(1-\frac{\upsilon_s}{\upsilon}\right)^{-1}$$

$$\nu = 1000\text{ Hz} \times [1 - 200\text{ m s}^{-1}/330\text{ m s}^{-1}]^{-1}$$

$$\simeq 2540\text{ Hz}$$

(2) The target is now the source (because it is the source of echo) and the rocket's detector is now the detector or observer (because it detects echo). Thus, $\upsilon_s = 0$ and υ_o has a positive value. The frequency of the sound emitted by the source (the target) is ν, the frequency intercepted by the target and not ν_o. Therefore, the frequency as registered by the rocket is

$$\nu' = \nu\left(\frac{\upsilon+\upsilon_0}{\upsilon}\right)$$

$$= 2540\text{ Hz} \times \frac{200\text{ m s}^{-1} + 330\text{ m s}^{-1}}{330\text{ m s}^{-1}}$$

$$\simeq 4080\text{ Hz}$$