

0) Find a vector and unit vector perpendicular to each of the vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$

$$\text{where } \vec{a} = 3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k} \quad \& \quad \vec{b} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$$

$$\vec{a} + \vec{b} = 3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$$

$$\therefore \vec{a} + \vec{b} = 4\mathbf{i} + 4\mathbf{j}$$

$$\vec{a} - \vec{b} = 3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k} - \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$$

$$\therefore \vec{a} - \vec{b} = 2\mathbf{i} + 4\mathbf{k}$$

Let \vec{c} be the vector perpendicular to $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ then $\vec{c} = (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$

$$\vec{c} = \begin{vmatrix} + & - & + \\ \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = \mathbf{i}(16 - 0) - \mathbf{j}(16 - 0) + \mathbf{k}(0 - 8)$$

$$\therefore \vec{c} = 16\mathbf{i} - 16\mathbf{j} - 8\mathbf{k}$$

$$|\vec{c}| = \sqrt{16^2 + 16^2 + (-8)^2} = \sqrt{256 + 256 + 64} = \sqrt{576} = 24$$

Let \hat{c} be the unit vector perpendicular to $(\vec{a} + \vec{b})$ and $\vec{a} - \vec{b}$

$$\text{then } \hat{c} = \frac{\vec{c}}{|\vec{c}|}$$

$$\hat{c} = \frac{16\mathbf{i} - 16\mathbf{j} - 8\mathbf{k}}{\sqrt{24}} = \frac{2\mathbf{i}}{3} - \frac{2\mathbf{j}}{3} - \frac{\mathbf{k}}{3}$$