

Find the area of parallelogram whose adjacent sides are  $2\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$  and  $\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ . Also find unit vector parallel to its diagonal.

$$\vec{a} = 2\mathbf{i} - 4\mathbf{j} + 5\mathbf{k} \quad \vec{b} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix} = \mathbf{i}\{12+10\} - \mathbf{j}\{-6-5\} + \mathbf{k}\{-4+4\} = 22\mathbf{i} + 11\mathbf{j}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(22)^2 + (11)^2 + 0} = \sqrt{242 + 121 + 0} = \sqrt{363}$$

$$\text{Area of parallelogram ABCD} = |\vec{a} \times \vec{b}| = \sqrt{363} \text{ sq. units}$$

$$\text{Diagonal } \vec{AC} = \vec{a} + \vec{b}$$

$$= 2\mathbf{i} - 4\mathbf{j} + 5\mathbf{k} + \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$$

$$\vec{AC} = 3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$$

$$|\vec{AC}| = \sqrt{3^2 + (-6)^2 + 2^2} = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$$

$$\text{Unit vector along } \vec{AC}, \quad \hat{\vec{AC}} = \frac{\vec{AC}}{|\vec{AC}|} = \frac{3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}}{7}$$

$$\text{Unit vector parallel to } \vec{AC} = \text{Unit vector along } \vec{AC} = \frac{3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}}{7}$$

