Q. For any two vectors $\vec{a}$ and $\vec{b}$ prove that $|\vec{a} \bullet \vec{b}| \le |\vec{a}| \bullet |\vec{b}|$

$$w.k.t.\ \cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|\ |\vec{b}|}$$

$$\frac{|\vec{a}.\vec{b}|}{|\vec{a}|\ |\vec{b}|} = |\cos\theta|$$

for all values of $\theta$, $-1 \le \cos\theta \le 1$

$$\therefore |\cos\theta| \le 1$$

$$\therefore \frac{|\vec{a}.\vec{b}|}{|\vec{a}|\ |\vec{b}|} \le 1$$

$$\therefore |\vec{a}.\vec{b}| \le |\vec{a}|\ |\vec{b}|$$

Q. For any two vectors $\vec{a}$ and $\vec{b}$ prove that $|\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$ (Triangle in equality)

$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2(\vec{a}.\vec{b}) + |\vec{b}|^2$$

$$\le |\vec{a}|^2 + 2|\vec{a}.\vec{b}| + |\vec{b}|^2 \qquad \because \vec{a}.\vec{b} \le |\vec{a}.\vec{b}|$$

$$\le |\vec{a}|^2 + 2|\vec{a}|\ |\vec{b}| + |\vec{b}|^2$$

From previous properties $|\vec{a}.\vec{b}| \le |\vec{a}||\vec{b}|$

$$\le \{|\vec{a}| + |\vec{b}|\}^2$$

$$\therefore |\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$$