Q. If $\vec{a} = 5i - j - 3k,\ \vec{b} = i + 3j - 5k$ then show that the vectors $\vec{a} + \vec{b}$ & $\vec{a} - \vec{b}$ are perpendicular.

$\vec{a} + \vec{b} = 5i - j - 3k + i + 3j - 5k = 6i + 2j - 8k$

$\vec{a} - \vec{b} = 5i - j - 3k - (i + 3j - 5k) = 4i - 4j + 2k$

consider $(\vec{a} + \vec{b}) . (\vec{a} - \vec{b}) = (6i + 2j - 8k).(4i - 4j + 2k) = 24 - 8 - 16 = 0$

$\therefore (\vec{a} + \vec{b}) . (\vec{a} - \vec{b}) = 0$ $\quad\quad$ $\therefore \vec{a} + \vec{b}$ is perpendicular to $\vec{a} - \vec{b}$

Q. If $\vec{a} = 2i + 2j + 3k,\ \vec{b} = -i + 2j + k$ and $\vec{c} = 3i + j$ and such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$ then find λ.

$\vec{a} + \lambda\vec{b} = 2i + 2j + 3k + \lambda(-i + 2j + k)$

$\therefore\ \vec{a} + \lambda\vec{b} = (2 - \lambda)i + (2 + 2\lambda)j + (3 + \lambda)k$

Given $\vec{a} + \lambda\vec{b}$ is $\perp^r$ $\vec{c}$.

$(\vec{a} + \lambda\vec{b}) . \vec{c} = 0$

$\{(2 - \lambda)i + (2 + 2\lambda)j + (3 + \lambda)k\} . \{3i + j\} = 0$

$6 - 3\lambda + 2 + 2\lambda = 0 \Rightarrow -\lambda + 8 = 0 \quad\quad \therefore \lambda = 8$