

If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then find $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

$$|\vec{a}| = 1, |\vec{b}| = 1, |\vec{c}| = 1$$

$$\vec{a} \cdot (\vec{a} + \vec{b} + \vec{c}) = \vec{a} \cdot \vec{0}$$

$$\vec{a}^2 + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = 0$$

$$\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = -\vec{a}^2$$

$$\vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\vec{b}^2$$

$$\vec{c} \cdot \vec{a} + \vec{a} \cdot \vec{b} = -\vec{c}^2$$

$$\text{Adding } 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -\vec{a}^2 - \vec{b}^2 - \vec{c}^2 = -1^2 - 1^2 - 1^2$$

$$2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -3$$

$$\therefore \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -3/2$$