Important properties and formulas

1. (a) Triangle law of vector addition $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

(b) Parallelogram law of vector addition : If ABCD is a parallelogram, then $\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$

(c) If $\vec{r_1} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\vec{r_2} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ then

$\vec{r_1} + \vec{r_2} = (x_1 + x_2)\hat{i} + (y_1 + y_2)\hat{j} + (z_1 + z_2)\hat{k}$

and $\vec{r_1} = \vec{r_2} \Leftrightarrow x_1 = x_2, y_1 = y_2, z_1 = z_2$

2. (a) $\vec{a}$ and $\vec{b}$ are parallel if and only if $\vec{a} = m\,\vec{b}$ for some non-zero scalar m.

(b) $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$ or $\vec{a} = |\vec{a}|\hat{a}$

(c) Associative law : $m(n\vec{a}) = (mn)\vec{a} = n(m\vec{a})$

(d) Distributive laws : $(m + n)\vec{a} = m\vec{a} + n\vec{a}$ and $n(\vec{a} + \vec{b}) = n\vec{a} + n\vec{b}$

(e) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then $m\vec{r} = mx\hat{i} + my\hat{j} + mz\hat{k}$

(f) $\vec{r}, \vec{a}, \vec{b}$ are coplaner if and only if $\vec{r} = x\vec{a} + y\vec{b}$ for some scalars x and y

3. (a) If the position vectors of the points A and B be $\vec{a}$ and $\vec{b}$ then,

(i) The position vectors of the points dividing the line AB in the ratio m :n internally and externally

are $\dfrac{m\vec{b} + n\vec{a}}{m + n}$ and $\dfrac{m\vec{b} - n\vec{a}}{m - n}$

(ii) Position vector of the middle point of AB is given by $\frac{1}{2}\left(\vec{a}+\vec{b}\right)$

(iii) $\overrightarrow{AB} = \vec{b} - \vec{a}$

(b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

(c) The points A,B,C will be collinear if and only if $\overrightarrow{AB} = m\,\overrightarrow{AC}$, for some non zero scalar m.

(d) Given vectors $x_1\vec{a} + y_1\vec{b} + z_1\vec{c}$, $x_2\vec{a} + y_2\vec{b} + z_2\vec{c}$, $x_3\vec{a} + y_3\vec{b} + z_3\vec{c}$, where $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors, will be coplanar if and only if $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0$

(e) **Method to prove four points to be coplanar :** To prove that the four points A,B,C and D are coplanar.Find the vector $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$ and then prove them to be coplanar by the metod of coplanarity i.e. one of them is a linear combination of the other two.

(f) $|\vec{a}+\vec{b}| \le |\vec{a}| + |\vec{b}|$

$|\vec{a}+\vec{b}| \ge |\vec{a}| - |\vec{b}|$

$|\vec{a}-\vec{b}| \le |\vec{a}| + |\vec{b}|$

$|\vec{a}-\vec{b}| \ge |\vec{a}| - |\vec{b}|$

Dot product of two vectors

(a) $\vec{a}.\vec{b} = ab\cos\theta$, where $0 \le \theta \le \pi$

(b) $\vec{a}.\vec{b} = a$ (Projection of $\vec{b}$ along $\vec{a}$)

(c) Projection of $\vec{b}$ along $\vec{a} = \dfrac{\vec{b}.\vec{a}}{|\vec{a}|}$

(d) The vector perpendicular to both $\vec{a}$ and $\vec{b}$ is given by $\vec{a}\times\vec{b}$

The unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ is given by $\hat{n} = \dfrac{(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$

(e) $\vec{a}\cdot\vec{b} = 0 \Rightarrow \vec{a} = 0$ or $\vec{b} = 0$

(f) Component of a vector $\vec{r}$ in the direction of $\vec{a}$ and perpendicular to $\vec{a}$ are $\left(\dfrac{\vec{r}.\vec{a}}{|\vec{a}|^2}\right)\vec{a}$ and

$\vec{r} - \left\{\dfrac{(\vec{r}.\vec{a})}{|\vec{a}|^2}\right\}\vec{a}$ respectively.

(g) If $\vec{a}$ and $\vec{b}$ are the non-zero vectors, then $\vec{a}.\vec{b} = 0 \Leftrightarrow \vec{a}\perp\vec{b}$

(h) $\cos\theta = \hat{a}.\hat{b} = \dfrac{\vec{a}.\vec{b}}{|a|.|b|}$

(i) $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

$\hat{i}.\hat{j} = \hat{j}.\hat{i} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = \hat{i}.\hat{k} = 0$

(j) If $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$ i.e. if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then

(i) $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

(ii) $\cos\theta = \dfrac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}$

(iii) $\vec{a}$ and $\vec{b}$ will be perpendicular if and only if $a_1b_1 + a_2b_2 + a_3b_3 = 0$

(iv) $\vec{a}$ and $\vec{b}$ will be parallel if and only if $\dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} = \dfrac{a_3}{b_3}$

Vector or cross product of two vectors

(a) The product of vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a} \times \vec{b}$.

$$\vec{a} \times \vec{b} = (|\vec{a}|\,|\vec{b}|\sin\theta)\hat{n}$$

(b) $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

(c) If $\vec{a} = \vec{b}$ or if $\vec{a}$ is parallel to $\vec{b}$, then $\sin\theta = 0$ and so $\vec{a} \times \vec{b} = 0$

(d) **Distributive laws :** $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$ and $(\vec{b} + \vec{c}) \times \vec{a} = \vec{b} \times \vec{a} + \vec{c} \times \vec{a}$

(e) The vector product of a vector $\vec{a}$ with itself is a null vector, i.e. $\vec{a} \times \vec{a} = \vec{0}$

(f) if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then

(i) $\vec{a} \times \vec{b} = (a_2b_3 - a_3b_2)\hat{i} + (a_3b_1 - a_1b_3)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}$

(ii) $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

(iii) $\sin^2\theta = \dfrac{(a_2b_3 - a_3b_2)^2 + (a_3b_1 - a_1b_3)^2 + (a_1b_2 - a_2b_1)^2}{(a_1^2 + a_2^2 + a_3^2)\,(b_1^2 + b_2^2 + b_3^2)}$

(g) If two vectors $\vec{a}$ and $\vec{b}$ are parallel, then $\theta = 0$ or π i.e. $\sin\theta = 0$ in both cases

$\because (a_1 b_2 - a_2 b_1)^2 + (a_2 b_3 - a_3 b_2)^2 + (a_3 b_1 - a_1 b_3)^2 = 0$

$\Rightarrow a_1 b_2 - a_2 b_1 = 0,\ a_2 b_3 - a_3 b_2 = 0,\ a_3 b_1 - a_1 b_3 = 0$

$$\Rightarrow \frac{a_1}{b_1} = \frac{a_2}{b_2},\quad \frac{a_2}{b_2} = \frac{a_3}{b_3},\ \frac{a_3}{b_3} = \frac{a_1}{b_1} \qquad \Rightarrow \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

Thus, two vectors $\vec{a}$ and $\vec{b}$ are parallel if their corresponding co,ponents are proportional.

(h) Area of the parallelogram ABCD = $|\overrightarrow{AB} \times \overrightarrow{AD}|$ or $\frac{1}{2}|\overrightarrow{AC} \times \overrightarrow{BD}|$

(i) Area of the triangle ABC = $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$