Example 8 Using vectors, prove that cos (A – B) = cosA cosB + sinA sinB.

Solution Let $\widehat{OP}$ and $\widehat{OQ}$ be unit vectors making angles A and B, respectively, with positive direction of x-axis. Then $\angle QOP = A - B$ [Fig. 10.1]

We know $\widehat{OP} = \overrightarrow{OM} + \overrightarrow{MP} = \hat{i}\cos A + \hat{j}\sin A$ and $\widehat{OQ} = \overrightarrow{ON} + \overrightarrow{NQ} = \hat{i}\cos B + \hat{j}\sin B$.

By definition $\widehat{OP} . \widehat{OQ} = \left|\widehat{OP}\right| \left|\widehat{OQ}\right| \cos (A\text{-}B)$

$$= \cos (A - B) \qquad \ldots (1) \qquad \left(\because \left|\widehat{OP}\right| = 1 = \left|\widehat{OQ}\right|\right)$$

In terms of components, we have

$\widehat{OP} . \widehat{OQ} = (\hat{i}\cos A + \hat{j}\sin A) . (\hat{i}\cos B + \hat{j}\sin B)$

$= \cos A \cos B + \sin A \sin B \qquad \ldots (2)$

From (1) and (2), we get

cos (A – B) = cosA cosB + sinA sinB.

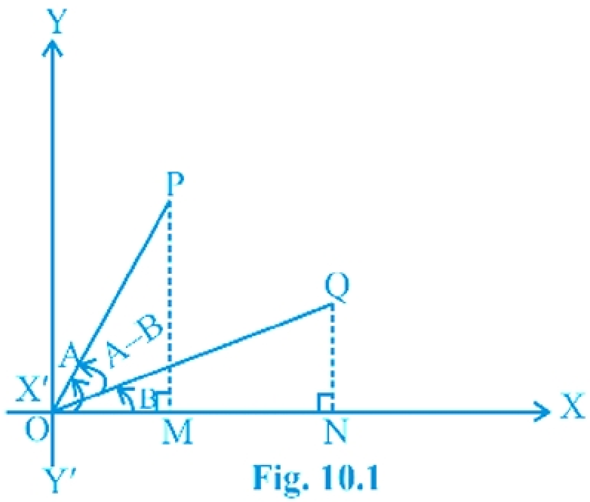

Fig. 10.1