

**Example 6** If  $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ ,  $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$  and  $\vec{c} = \hat{i} + 3\hat{j} - \hat{k}$ , find  $\lambda$  such that  $\vec{a}$  is perpendicular to  $\lambda\vec{b} + \vec{c}$ .

**Solution** We have

$$\begin{aligned}\lambda\vec{b} + \vec{c} &= \lambda(\hat{i} + \hat{j} - 2\hat{k}) + (\hat{i} + 3\hat{j} - \hat{k}) \\ &= (\lambda + 1)\hat{i} + (\lambda + 3)\hat{j} - (2\lambda + 1)\hat{k}\end{aligned}$$

Since  $\vec{a} \perp (\lambda\vec{b} + \vec{c})$ ,  $\vec{a} \cdot (\lambda\vec{b} + \vec{c}) = 0$

$$\Rightarrow (2\hat{i} - \hat{j} + \hat{k}) \cdot [(\lambda + 1)\hat{i} + (\lambda + 3)\hat{j} - (2\lambda + 1)\hat{k}] = 0$$

$$\Rightarrow 2(\lambda + 1) - (\lambda + 3) - (2\lambda + 1) = 0$$

$$\Rightarrow \lambda = -2.$$