The radius of the base of a cone is increasing at the rate of 3 cm/min and the altitude is decreasing at the rate of 4 cm/min. The rate of change of lateral surface when the radius is 7 cm and altitude is 24 cm is

a. $108\pi\ cm^2/min$ **b.** $7\pi\ cm^2/min$

c. $27\pi\ cm^2/min$ **d.** none of these

a. Let r, l, and h denote, respectively, the radius, slant height, and height of the cone at any time t. Then,

$l^2 = r^2 + h^2$

or $2l\dfrac{dl}{dt} = 2r\dfrac{dr}{dt} + 2h\dfrac{dh}{dt}$

or $l\dfrac{dl}{dt} = r\dfrac{dr}{dt} + h\dfrac{dh}{dt}$

$= 7 \times 3 + 24 \times (-4)$ $\left[\because \dfrac{dh}{dt} = -4 \text{ and } \dfrac{dr}{dt} = 3\right]$

$= -75$

Where $r = 7$ and $h = 24$, we have

$l^2 = 7^2 + 24^2$ $[\because l^2 = r^2 + h^2]$

or $l = 25$

$\therefore l\dfrac{dl}{dt} = -75$ or $\dfrac{dl}{dt} = -3$

Let S denote the lateral surface area. Then

$$\frac{dS}{dt} = \frac{d}{dt}(2\pi rl) = 2\pi\left\{\frac{dr}{dt}l + r\frac{dl}{dt}\right\}$$

$$= 2\pi\{3 \times 25 + 7 \times (-3)\}$$

$$= 108\pi\ cm^2/min$$