Rate Measure

(1) Area of circle = $A = \pi r^2$

Rate of change of area = $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

(2) Volume of Sphere $V = 4/3 \pi r^3$

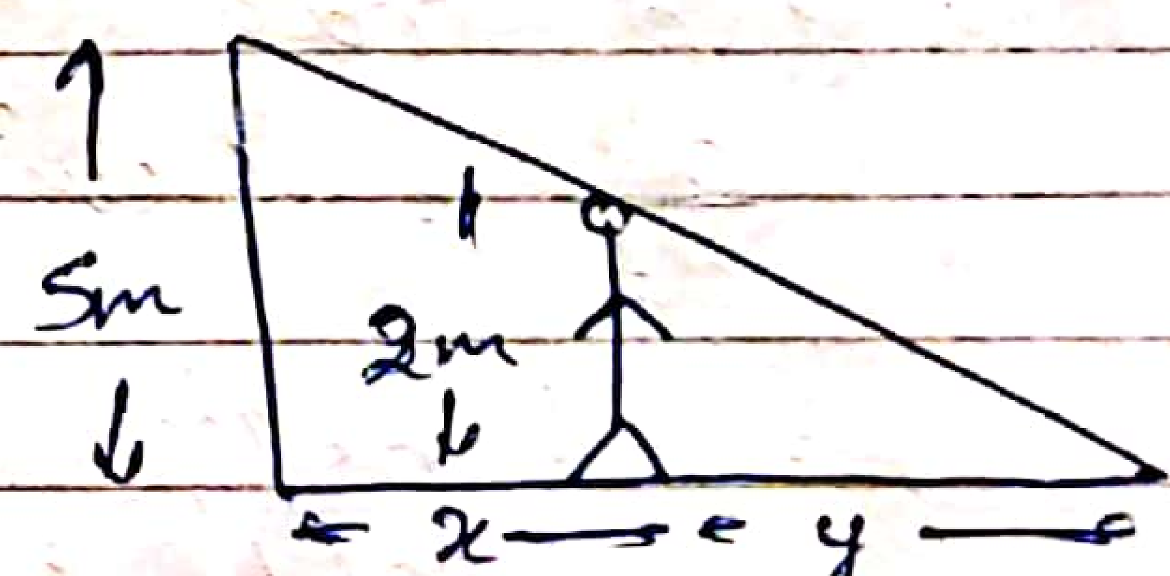
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

(3) Vol. of cylinder, $V = \pi r^2 h$

$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + 2\pi r h \frac{dr}{dt}$

(4) Vol. of cone ; $V = 1/3 \pi r^2 h$

Q. A man 2m high is walking away from a lamp post of 5m height at 6 m/min speed find rate at which shadow increases?



$\frac{x+y}{y} = \frac{5}{2}$

$2x + 2y = 5y$

$2x = 3y$

$2\frac{dx}{dt} = 3\frac{dy}{dt}$

$2 \times 6 \text{ m/min} = 3\frac{dy}{dt}$

$\frac{dy}{dt} = 4$ ~~mints m/min~~

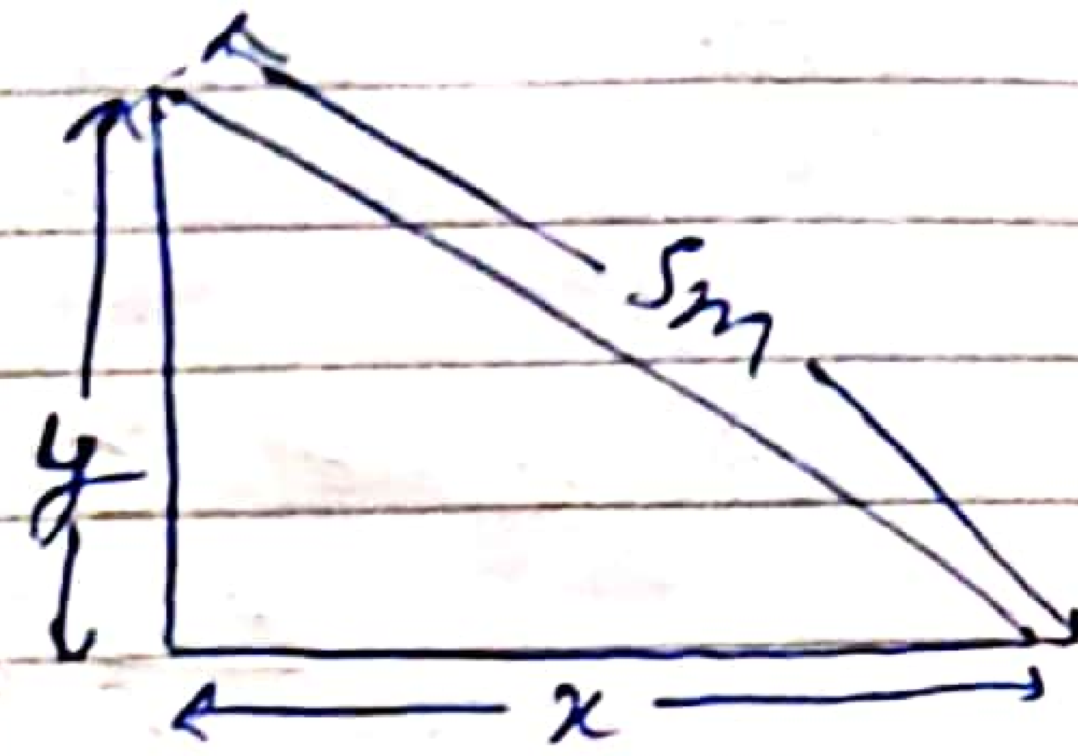
Shadow is increases at the rate of 4 m/min

Q. A ladder 5m long is leaning against the wall bottom of the ladder is pulled away at 2m/s. How fast the height of the ladder of the wall is ~~increasing~~ decreasing when right ~~ladder side~~ is 4m.

$x^2 + y^2 = 25$

$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = ?$



When $y = 4$, $x = 3$

$3 \times 2 + 4\frac{dy}{dt} = 0$

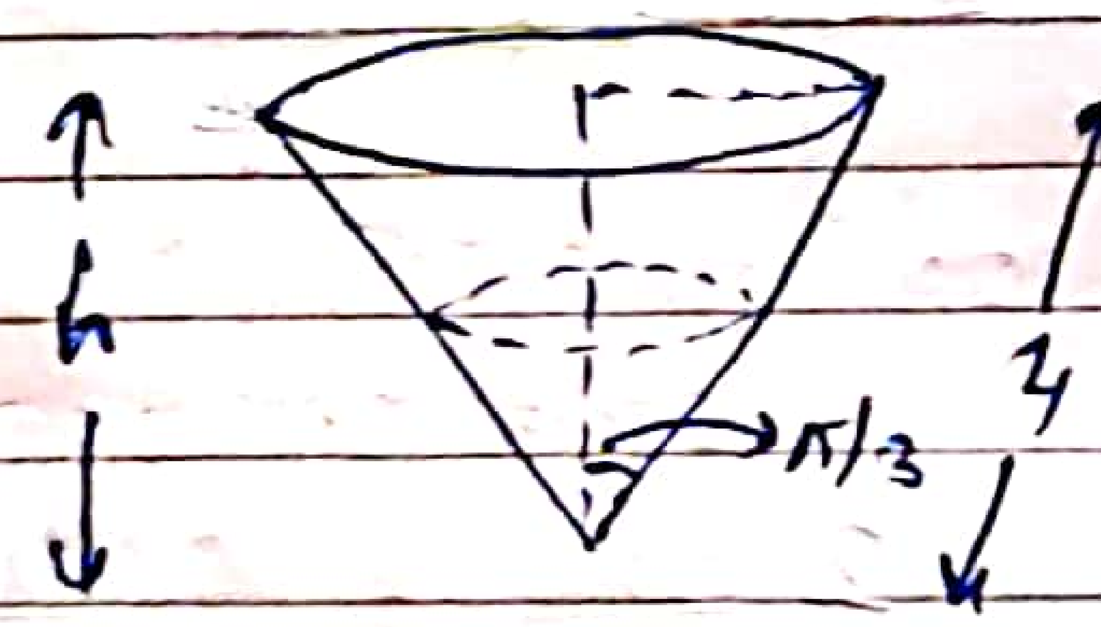
$\frac{dx}{dt} = 2m/s$

$\frac{dy}{dt} = -1.5\ m/s$

So, height of ladder decreasing at the rate of 1.5m/s.

Q. Water is running out from a conical vessel having semivertical angle $\pi/3$ at the rate $4cm^3/s$. When slant of vessel is 4cm, then find rate of change of slant height?

$\frac{dV}{dt} = 4\ cm^3/s$



$h = l\cos\pi/3$
$= 4\cos\pi/3 = 2\ cm$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{\sqrt{3}}{2}l\right)^2 \times \left(\frac{l}{2}\right)$

$V = \frac{\pi}{8}l^3$

$\frac{dV}{dt} = \frac{3\pi}{8}l^2\frac{dl}{dt}$

$4 = \frac{3\pi}{8} \times 4^2 \times \frac{dl}{dt}$

$\frac{dl}{dt} = \frac{2}{3\pi}\ cm/s$

Q. Sand is pouring from a pipe at rate of 12 cm³/s & forming a cone at ground in such a way that height of cone is 1/6th of its radius.

How fast the height of cone is increasing when height is 4 cm.

$$h = \frac{r}{6} \Rightarrow r = 6h$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (6h)^2 h$$

$$V = 12\pi h^3$$

$$\frac{dV}{dt} = 36\pi h^2 \frac{dh}{dt}$$

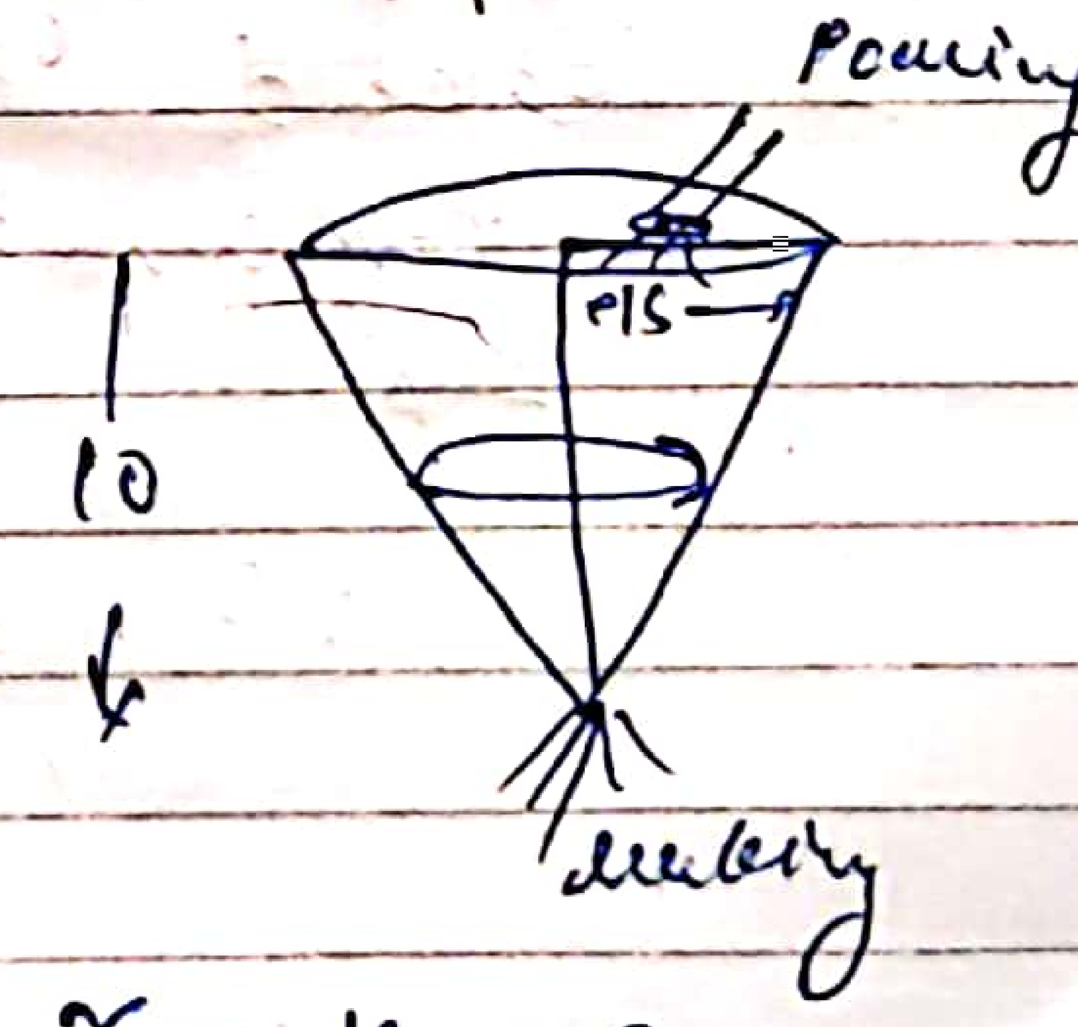
$$\cancel{12} = \overset{3}{\cancel{36}} \times \pi \times 4^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{48\pi} \text{ cm/s}$$

Q A water tank has a shape of right circular cone with its vertex down, its altitude is 10 cm & radius is 15 cm. Water leaks from bottom at const. rate of 1 cm³/s & water is poured at a const. rate of c cm³/s. Compute c, so that water level is rising at the rate of 4 cm³/s at an instant when water 2 cm deep.

leak – 1 cm³/s   pour → c cm³/s

$$\frac{dV}{dt} = c - 1 \text{ cm}^3/\text{s}$$



$$\frac{r}{h} = \frac{15}{10} = \frac{3}{2}$$

$$V = \frac{\pi}{3} r^2 h$$

$$= \frac{\pi}{3}\left(\frac{3h}{2}\right)^2 h$$

$$V = \frac{3\pi}{4} h^3$$

$$\frac{dV}{dt} = \frac{9\pi}{4} h^2 \frac{dh}{dt}$$

$C - 1 = \frac{9\pi}{\cancel{\pi}} \times (2)^2 \times \cancel{\pi}$

$C = 36\pi + 1 \quad cm^3/s$

Q. A spherical ballon is filled with ~~4500 cm³ ft/H~~ $4500\pi m^3$ of the gas. If a leak causes the gas to escape at rate $72\pi\ m^3/min$. Find the rate at which radius of the ballon decreases 49 min after the leakage began is ?

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$72\pi = 4\pi r^2 \frac{dr}{dt}$

$\cancel{72}^{18}\cancel{\pi} = \cancel{4}\cancel{\pi} \times 15 \times 15 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{18^2}{\frac{\cancel{18} \times 18}{5 \times 5}} = 0.08\ cm/min$

$V = \frac{4}{3}\pi r^3$

$\cancel{4500}^{1125} \cancel{\pi} = \frac{4}{3}\cancel{\pi} r^3$

$r = 15\ cm$

Gas is leaking at rate of $72\pi\ m^3/min$

Gas leaked in 49 min $= 72\pi \times 49\ m^3$

has remained $= 4500\pi - 72\pi \times 49$

$\frac{4}{3}\cancel{\pi} r^3 = 972\cancel{\pi}$

$r^3 = 729$

$r = 9\ cm$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\cancel{72}^{2}\pi = \cancel{4}\cancel{\pi} \times 9^2 \frac{dr}{dt}$

$\frac{dr}{dt} = 2/9\ m/min$

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